ON THE NATURE OF THINGS

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An Introduction to the Universe:

The future of humanity in our hostile universe is uncertain. Our survival as a species depends on if we are able to spread out to inter-stellar space, which means creating new space-fairing technologies. Without this effort of spreading our population throughout the solar system's planets and moons, humanity is sure to go extinct within the next thousand years, from all sorts of disasters. In comparison to the 13.8 billion year age of the universe, that is a cosmic blink of the eye.

These disasters include man made threats like climate change, nuclear war, biotech or AI, or even natural disasters like super-volcanoes, or asteroid strikes. Then there are unknown disasters that would occur without warning, because our science was not up to the task to identify them. If one cannot identify the problem, one cannot solve it. That is why in science, if we have a theory, we try to disprove it by any means necessary, such that we spot the problems in the math that need to get fixed to agree with reality.

Scientific advances in the last few hundred years have made it possible for new technology, improving the quality of life as a byproduct, and paving the way for new discoveries in science. This included Newton's principia in 1687 which spurred the industrial revolution, and Maxwell's 1865 theory of electricity, magnetism, and light, which allowed the electrification of our homes, and by including quantum physics has made it possible for the wonders of the modern age. The very book I am writing this on, depends on computers, which were made possible by the transistor and quantum physics of semi-conductors. But most importantly, the technology that allows travel into deep space, will require new physics that is beyond our current reach, specifically a theory of everything.

The usual channels of scientific communication—specialized papers, review articles, textbooks—are built for incremental results or for theories that already sit inside an accepted framework. A proposal that recovers the three pillars of modern physics from a single classical construction, supplies the explicit tetrads and wave equations, and includes a working numerical engine cannot be carried by those forms without being misread. This book is therefore written in a different register.

It is a systematic derivation (the CGH domain equations, the Kerr–Schild tetrad and its spin connection, the multi-particle solutions, the embedding of the Standard Model Lagrangian). It is also an urgent document, because the civilizational clock does not wait for the next incremental step. The extreme claim is matched by extreme evidence: every central construction is given in sufficient detail to be simulated, and the appendix contains a complete, runnable proof-of-concept implementation. The reader is invited to treat the mathematics and the code as the primary content; the surrounding prose exists only to make that content legible and to state why the work was done.

Our survival as a species should motivate us to do better and take such an ultimate theory seriously. In fact, it is not as bad as it seems. Everything in the universe

can be explained via the laws of physics, and these laws are well tested across a broad domain of research areas. Today, there are two main theories of physics. One is general relativity, the other is quantum field theory. General relativity is about gravity, space, and time, and has consequences like the big bang, black holes, and gravitational waves. The quantum field theory that applies to reality is known as the standard model of particle physics, and it describes everything else except gravity. And from the 96 types of matter waves, to the 28 types of force waves, it is a quantum theory, which means that each and every ripple in these waves is by definition a particle. An electron with a left handed spin is a discrete ripple in one of the 96 matter waves, whereas a photon with a clockwise polarization is yet another discrete ripple, this time in one of the 28 force waves, that we call the electro-magnetic field. When both general relativity and quantum field theory are glued together, it becomes a daunting task, maybe even impossible. So what now, we have two great theories, but their unification into a complete theory is off limits. We are very close, like a millimeter from the finish line, but once we cross it, we will know exactly where we went wrong, and be able to complete the theory that future generations depend on.

I personally know the stakes, and have spent my whole life learning about black holes, and unification attempts. Maybe the theory wasn't supposed to be unified after all, at least not in the way as described by string theorists or quantum gravity experts. We should take what we know seriously enough and be careful about assuming the answer before a reveal. And without a reveal, there is even less evidence to go by. This means that assumptions of things like strings vibrating in higher dimensions obeying super-symmetry, or even that gravity being quantum mechanical require experiments beyond anything we can even concieve of.

This book is written in the form required by its content: a fully explicit classical construction, supplied with working code, because no narrower scientific genre can carry both the claim and the evidence. It lays out the physics of our universe in hopes that one day, somebody can test it and say yes or no, to if we indeed have our final theory of everything. With somebody like myself studying from a very young age to find a theory of everything is rare, and made even more rare with my autism diagnosis. Maybe something new is required after all these unification attempts. And that is exactly what I am doing for this book. After all, by sidestepping quantum gravity might be the only way to solve it, and with no extra assuptions compared to existing physics, this is indeed the simplest theory of everything canidate in existence today. I will walk you through my reasoning behind the theory of everything I uncovered, and in so doing, show you why it is important to our survival as a species.

This book is also a modern reworking inspired by *De Rerum Natura* by Lucretius. Although much ancient philosophical literature has been lost to time, Lucretius' poem itself survives in an incomplete manuscript tradition, having narrowly escaped disappearance before its rediscovery in the early fifteenth century. Written over two

millennia ago, the work presents a poetic synthesis of Epicurean philosophy, itself rooted in Democritus' atomism: the doctrine that all things consist of atoms moving in a void. While the destruction of ancient libraries—most notably those associated with Alexandria—contributed to the loss of many classical texts, Lucretius' work endured through later copying rather than continuous transmission. Since antiquity, science has independently rediscovered and vastly expanded upon several of the core ideas anticipated by ancient atomism. However, modern physics has profoundly revised these notions.

Today, two principal theoretical frameworks dominate fundamental science: the standard model of cosmology and the standard model of particle physics. Although neither corresponds directly to the atomism of antiquity, both echo its central insight—that nature is governed by universal principles acting on fundamental constituents within spacetime. This work revisits Lucretius' vision in light of modern scientific understanding, offering a contemporary reinterpretation rather than a reconstruction of the ancient doctrine.

This means that the book here is a work of science and natural philosophy. Its purpose is to describe the physical universe using empirically grounded models, mathematical structure, and testable principles. In doing so, it necessarily limits itself to questions that can be addressed by observation, experiment, and logical consistency.

Claims concerning morality, ultimate purpose, divine intention, and the afterlife fall outside the scope of scientific inquiry as practiced here. Such claims are neither affirmed nor denied by this work; rather, they are recognized as belonging to philosophical, theological, or cultural traditions that operate according to different standards of meaning and justification. Readers should therefore not interpret the absence of supernatural explanations as a polemic against faith, but as a methodological boundary intrinsic to science itself.

The cosmological narrative presented in this book—including its discussion of the Big Bang—serves as a scientifically inspired account of origins, not a metaphysical or theological one. It is offered as a model of physical genesis grounded in contemporary physics, without making claims about ultimate causation, divine agency, or moral law.

Readers from religious traditions are invited to approach this work in the spirit of dialogue rather than conflict: as an exploration of how the universe behaves, not a declaration of how one ought to live or what one must believe. Where science is silent, this book remains silent. Where science speaks, it does so without intent to offend, persuade, or replace personal faith.

The book is dedicated to both our future descendants and to the scientists throughout history. Science is a way of thinking much more than it is a body of knowledge. Sure the mathematics is important in its own right, such as for a whole plethora of practical applications. But, it is in the way that scientists think, that has gotten us to the theoretical frameworks we have today. They go out into nature and test

their ideas. Keep the ideas that pass the test, reject the ideas that fail the test. Refine your ideas and testing methods until they match reality.

The approach developed here is a classical-field formulation that has not been the dominant research direction. Whether it is correct or not, the explicit constructions (Kerr-Schild tetrads, the CGH reductions, the embedding of the Standard Model Lagrangian) are concrete enough to be tested, simulated, and refined.

Chapter 1: Extreme Scales of Time, Space, and Matter

We begin our epic journey with a very unlikely topic: The immensity of the cosmos that surrounds us. Most of us give the universe a second thought. They focus on what is right in front of them, and are selfish to other people's perspectives. If they had expanded their horizons we wouldn't be having this discussion. There is only one universe and that is all there is to existence. Everything that has ever existed or will ever exist, including everyones lived experiences, is a tiny fraction of the universe.

To get to know our shared universe, it is necessary to know that not everybody will be able to understand it, or why it is important. And that is totally fine. This book is not for everybody. It for the person who is interested in deep space and deep time, because he or she already knows the existential risks of not taking the challenge. After all if everyone were the same, life would be boring. The same is true of of the whole universe. Most people in the universe are interested in playing small. But a select few in the same universe are what makes reality interesting, and if you are one of these select few that enjoys the vastness of the universe, then stick with me here for the book. I will assure you, it will be worth it.

Extreme scales puts' our universe and us into perspective. We are nothing compared to the universe. Humans are only a recent arrival on Planet Earth, a tiny but old speck of dust in a vast and ancient expanse of nothingness. We seem insignificant. True physically, but mentally? We might be the only species that can ever confront the universe, and say I understand how everything works fundamentally. Most if not all of nature is dead weight. We as human beings that can think, feel, create, and so forth, are the only part of the universe that is aware of itself. Following the same laws that make everything possible, extreme scales is about widening that awareness away from one's self into alignment with the rest of the reality.

Therefore as the first chapter to this book, it not only puts humanity into perspective, but also shows why saving humanity in such a bleak universe is not impossible, but already done. If you were to follow me to the end of the book, you would be one of the few to know the answer to how everything works. Therefore knowing extreme scales supplies you with the mental tools needed to overcome the vastness of the universe.

But a great place to start is to ask a decepively simple question, not what is science, not even what is theoretical physics on which the rest of science depends. We are to begin with the simplest possible question, one that all theoretical physics depends on, how does one measure the void, otherwise put, how can something come from nothing?

Section 1.1: What is it?

What is it? What is time? What is space? What is matter? To answer these questions, I have three analogies that use the concepts of time, space, and matter, that explain what they are. Time is a single dimension that can be experienced in one direction and not backwards. Space does not have that limitation, but is three dimensional, making calculations much more difficult. Matter is not a dimension you live in, but a dimension that you can count. Mass is the amount of matter. So there we have it, matter lives in a 3D space, which evolves in time. Let me elaborate some more on these distinctions with these analogies.

For time, think of a huge 1D line. We can break up the line into segments of equal elapsed time. Each vertex connects one segment to the following segment. We experience time, one vertex after another. And we can number them. If the current vertex number is time t, the next vertex is t + 1, and the previous vertex is t – 1. Clearly, if time t is now, then the only time that is accessable, is t + 1. All of the other times are not, and don't seem to exist.

If one were to write any game, one first builts a clock that tells the game what snapshot you are in. By placing one snapshot after another, you build the game frame by frame into a video. In code we litterally compute frame t, and then ask "what will the players see after 1 segment of evolution?", after we compute frame t+1, we relabel t+1 as t, and continue. Repeat this process and t seems to count and increase from 0. But we are only counting one at a time, not all at once. This is a game hack that increases performance.

About space, its a 3D continuum. How we sub-divide it is up to us. I say this, because I am about to introduce the 5 platonic solids. Like we did for time, which is a 1D line, we subdivided the line into equal segments connected by vertices. We can do the same thing for space. Space is 3D, so lets build up to 3D by talking about 1D and 2D spaces first.

First we start in 1D. The line segment with two vertices X=0 and X=1, one can draw a perfectly straight line between them. This is the only shape in one dimension. Luckily it repeats. Like the 1D time-line situation, one can translate one line segment unto the next, and this process can repeat indefinitely.

Next up is 2D. Instead of starting from scratch, one can use the points and lines from earlier. In 1D space one can only translate X to X+dX. But in 2D, one has more freedom. We have a new Y dimension that can also be translated from Y+dY. And lastly we have a single 2D rotation that mixes the X and Y axes into each-other. This rotation is given by the following equation, with sines and cosines of an angle theta:

$X = X*\cos(\theta) - Y*\sin(\theta)$

$Y = Y*\cos(\theta) + X*\sin(\theta)$

So with these constraints, one can move the vertices of the line segments into place with other vertices. The only possible shapes in a 2D Euclidean space like this, are

the regular polygons. The great mathematician Euler came up with a simple formula for these polygons, using the same rotation formula from earlier. One just repeats the same rotation over and over until one arrives back at the starting point, and yes that is where the number pi comes from. It is a 2*pi rotation that brings the identity rotation, or any multiple thereof.

For a single sided polygon, its actually just a single vertex connected to itself. Yes technically its a line segment, its just that its two vertices coincide at N=1.

$(X,Y) = (\cos(2\pi N),\sin(2\pi N))$

for N=1 1 0

For a two sided polygon, it looks like a single vertex that has vertices from (1,0) to (-1,0), but there is another one that does the opposite closing the circle. Now the two segments have coincided. Note the half way turn as compared to previously. If a full turn is 360 degrees, we have done two 180 degree rotations.

$(X,Y) = (\cos(2\pi N/2),\sin(2\pi N/2))$

for N=1 -1 0

for N=2 1 0

For a equilateral triangle, we have something interesting. With a rotation angle of 120 degrees, we have a shape that repeats and tiles in 2D space. This is our first example of three 2D tilings. This triangle will later be used for building three of the five platonic solids. By gluing 5 triangles together per vertex, a whopping 20 triangles makes an Icosahedron. By gluing 4 triangles per vertex, another 8 triangles can make us an Octahedron. And again, by gluing 3 triangles per vertex, another 4 triangles can make a Tetrahedron. However, the 2D tiling is what one gets when gluing 6 triangles per vertex. Here are the vertex positions for such a triangle.

$(X,Y) = (\cos(2\pi N/3),\sin(2\pi N/3))$

for N=1 -1/2 sqrt(3)/2

for N=2 -1/2 -sqrt(3)/2

for N=3 1 0

For a square, we again have something interesting. This time, we have a rotation angle of 90 degrees, we also have a shape that repeats and tiles in 2D space. This is our second example of the three 2D tilings. One platonic solid, the cube is made from gluing 3 squares together per vertex and requires 6 of these squares. Unlike the tiling which requires 4 squares per vertex. Here are the vertex positions for such a square.

$(X,Y) = (\cos(2\pi N/4),\sin(2\pi N/4))$

for N=1 0 1

for N=2 -1 0

for N=3 0 -1

for N=4 1 0

For a regular pentagon, there is more interesting stuff. It is built from 5 succesive 72 degree rotations. Up to now, we had building blocks that tile in 2D space.

For the first time, the pentagon, it doesn't tile. However it isn't completely useless, in that by gluing together 3 pentagons per vertex one arrives at the most complex 3D platonic solid with 12 pentagons, the Dodecahedron. Here are the vertex positions for such a pentagon.

$(X,Y) = (\cos(2\pi N/5),\sin(2\pi N/5))$

for N=1	(sqrt(5)-1)/4	sqrt((5+sqrt(5))/8)
for N=2	-(sqrt(5)+1)/4	sqrt((5-sqrt(5))/8)
for N=3	-(sqrt(5)+1)/4	-sqrt((5-sqrt(5))/8)
for N=4	(sqrt(5)-1)/4	-sqrt((5+sqrt(5))/8)
for N=5	1	0

Lastly, for a regular hexagon, there is not much else interesting stuff. Nonetheless, it is built from 6 rotations of 60 degrees. Unlike the previous polygons, hexagons do not form higher dimensional platonic solids. However hexagons are the last of the polygons to tile. By gluing 3 hexagons together per vertex, one gets the hexagonal tiling, the third of the three examples of 2D tilings. Here are the vertex positions for such a hexagon.

$(X,Y) = (\cos(2\pi N/6),\sin(2\pi N/6))$

for N=1	1/2	sqrt(3)/2
for N=2	-1/2	sqrt(3)/2
for N=3	-1	0
for N=4	-1/2	-sqrt(3)/2
for N=5	1/2	-sqrt(3)/2
for N=6	1	0

For higher and higher order "n" regular polygons, the rotations one uses is 360 degrees over n, so while n gets small, the polygons approach a circle in the limiting case where n goes to infinity. Yes thats right, there are an infinite number of regular polygons, but only 5 platonic solids. The only shapes that tile and repeat are n=3, n=4, n=6, and no other. And the only shapes that form higher dimensional platonic solids, are n=3, n=4, and n=5. So all polytopes with 7 or more vertices, will not tile and not form any complex structures. A waste of space. Here is the general form for a rotation that rotates one vertex of any regular polygon on-to the next one.

$(X,Y) = (\cos(2\pi/n),\sin(2\pi/n))$

Now we move on to 3D. The degrees of freedom include 3 rotation angles, and 3 translations for the coordinates X,Y,Z. The rotation angles are like the x,y pair we had earlier, but are shifted to y,z and z,x. They do not commute, so for the first time, we don't have a simple formula for the 5 platonic solids. Instead we will have to build them, and analyze them geometrically and then algebraically. We first define a number known as the golden ratio. Its useful for the last two platonic solids. The Dodecahedron and Icosahedron.

$\varphi = (1+\text{sqrt}(5))/2$

Here are the vertices for all 5 platonic solids.

Tetrahedron:
(1, 1, 1)
(1, −1, −1)
(−1, 1, −1)
(−1, −1, 1)

Octahedron:
(±1, 0, 0)
(0, ±1, 0)
(0, 0, ±1)

Cube:
(±1, ±1, ±1)

Icosahedron:
(0, ±1, ±φ)
(±1, ±φ, 0)
(±φ, 0, ±1)

Dodecahedron:
(±1, ±1, ±1)
(0, ±1/φ, ±φ)
(±1/φ, ±φ, 0)
(±φ, 0, ±1/φ)

In fact we could keep going. In 4D Euclidean space, with 4 spacial dimensions, we have 6 polytopic solids. In fact, if we took fractional "n" for our polygons we'd arrive at an infinite number of regular star polygons. In 3d, we would have 4 such star polyhedra. They contain a pentagram, a polygon with n = 5/2. In 4D Euclidean space again, 10 such star polytopes exist. Going to 5D space, there only exist 3 convex solids and nothing else. We can do this, because even though anything beyond 3D is not possible in physics. It is still possible in the math. Here is a short list of the polytopes, polyhedra, and their star variations.

2D:
{n}
{n1/n2}

2D Tilings:
{6,3}

{4,4}
{3,6}

3D:	Faces:	Edges:	Vertices:
{3,3}	4{3}	6	4{3}
{4,3}	6{4}	12	8{3}
{3,4}	8{3}	12	6{4}
{5,3}	12{5}	30	20{3}
{3,5}	20{3}	30	12{5}

3D:	Faces:	Edges:	Vertices:
{5/2,5}	12{5/2}	30	12{5}
{5,5/2}	12{5}	30	12{5/2}
{5/2,3}	12{5/2}	30	20{3}
{3,5/2}	20{3}	30	12{5/2}

3D Cubic Honeycomb:
{4,3,4}

4D:	Cells:	Faces:	Edges:	Vertices:
{3,3,3}	5{3,3}	10{3}	10{3}	5{3,3}
{4,3,3}	8{4,3}	24{4}	32{3}	16{3,3}
{3,3,4}	16{3,3}	32{3}	24{4}	8{3,4}
{3,4,3}	24{3,4}	96{3}	96{3}	24{4,3}
{5,3,3}	120{5,3}	720{5}	1200{3}	600{3,3}
{3,3,5}	600{3,3}	1200{3}	720{5}	120{3,5}

4D:	Cells:	Faces:	Edges:	Vertices:
{3,5,5/2}	120{3,5}	1200{3}	720{5/2}	120{5,5/2}
{5/2,5,3}	120{5/2,5}	720{5/2}	1200{3}	120{5,3}
{5,5/2,5}	120{5,5/2}	720{5}	720{5}	120{5/2,5}
{5,3,5/2}	120{5,3}	720{5}	720{5/2}	120{3,5/2}
{5/2,3,5}	120{5/2,3}	720{5/2}	720{5}	120{3,5}
{5/2,5,5/2}	120{5/2,5}	720{5/2}	720{5/2}	120{5,5/2}
{5,5/2,3}	120{5,5/2}	720{5}	1200{3}	120{5/2,3}
{3,5/2,5}	120{3,5/2}	1200{3}	720{5}	120{5/2,5}
{3,3,5/2}	600{3,3}	1200{3}	720{5/2}	120{3,5/2}
{5/2,3,3}	120{5/2,3}	720{5/2}	1200{3}	600{3,3}

5D and beyond:

{3,3,…,3,3}	6	15	20	15	6
{4,3,…,3,3}	10	40	80	80	32
{3,3,…,3,4}	32	80	80	40	10

etc…

Here is the symbol for the 3D Cubic Honeycomb:
{4,3,4}
and its vertices. It tiles a 3D cube forever into the 3D Euclidean space. This is the unique tiling of our physical space we live in.
(±N, ±N, ±N)

Now that we have both time as a restricted 1D tiling, and space as an unrestricted 3D tiling(the 3D Cubic Honeycomb), its time to move on to mass as the amount of matter. Since mass is not a physical dimension, as it only measures or counts things, we will leave it alone, and talk about something that will be useful to it. Divisability.

Say we have N marbles. And want to group them with equal subsets. How would you do this. You will divide the numbers of marbles into chunks, until you got down to the prime numbers.

0	1,2,3,4,5,6,etc…
1	1
2	1,2
3	1,3
4	1,2,4
5	1,5
6	1,2,3,6
7	1,7
8	1,2,4,8
9	1,3,9
10	1,2,5,10
11	1,11
12	1,2,3,4,6,12
13	1,13
14	1,2,7,14
15	1,3,5,15
16	1,2,4,8,16
17	1,17
18	1,2,3,6,9,18
19	1,19
20	1,2,4,5,10,20
21	1,3,7,21
22	1,2,11,22
23	1,23
24	1,2,3,4,6,8,12,24
25	1,5,25

26 1,2,13,26
27 1,3,9,27
28 1,2,4,7,14,28
29 1,29
30 1,2,3,5,6,10,15,30
31 1,31
32 1,2,4,8,16,32

For 32 marbles, of identical mass. One can split it into 32 marbles, 2 groups of 16 marbles, 4 groups of 8 marbles, 8 groups of 4 marbles, 16 groups of 2 marbles, and 32 marbles again. Therefore prime numbers are the "atoms" of numbers. They tell you how identical marbles can group together in different amounts to produce an entire universe made of nothing but marbles, living on the vertices of our 3D Euclidean space, evolving one frame after another. Now that we have a grasp on the basics of time, space, and matter, its time to measure it.

Section 1.2: Units of Measurement

The scale of the universe is far beyond human imagination. To even grasp it, we build scale models of objects in the universe and bring them to the everyday scale so we can understand them. There are three ways to do this, and each way or type of scale is defined by units of measurement. There are elapsed times, distances in space, and amounts of mass.

Before we begin, let's first understand orders of magnitude. Take one million marbles, as an example. We have a unit of measurement, the number of marbles. I could write this as "1,000,000 marbles", or as "1 * 10^6 marbles", or even as "one million marbles". If I define this as a mega-marble it means that I have now simplified this setup, by introducing a new unit of measurement. Since numbers can get very huge very fast, it becomes useful to define a new system of prefixes for our numbers. We have already met one prefix, mega, which means million. There are others too. A marble is 1 marble. A kilo-marble is 1,000 marbles. A mega-marble is 1,000 kilo-marbles. A gigamarble is 1,000 mega-marbles. A tera-marble is 1,000 giga-marbles. A peta-marble is 1,000 tera-marbles. This pattern continues on, by multiplying by one thousand every time. If we divided by a thousand instead of multiplying, we arrive at the following units of, milli-marbles, micro-marbles, nano-marbles, pico-marbles, and femto-marbles.

Now with this in mind, imagine two marbles floating in the void. Time runs at the same universal rate for both marbles. Both marbles each maintain their motion, as they both independently travel in straight lines with a constant speed. The two marbles both have definite positions as well as definite velocities. This all makes perfect sense. In order to define what is going on, we introduce a system of units that measure time, space, and matter.

Time controls the evolution of the marbles. This is because, since the marbles are moving, their positions change over time. At every moment or snapshot of time, there is a unique configuration of our marbles. To specify which configuration we are talking about, it becomes useful to introduce a clock that measures which snapshot we are in. Playing each snapshot back in sequence, moment after moment, we recover the evolution of the scene. This is just like the pages in a flip book or the frames of film in a movie projector. The clock time is universal and we count the frames with a new unit of measurement, the second.

The Second is defined as follows: A full day by definition lasts 86,400 seconds. A minute is 60 seconds. An hour is 60 minutes. A day is then 24 hours. One week is 7 days. One month is 4 and 1/3 weeks. A year is 12 months. A decade is 10 years. A generation is 3 decades. Similarly, a kilosecond is one thousand seconds, and a millisecond is a thousandth of a second.

Space is measured by using a three dimensional coordinate grid. The X dimension runs from left to right. Next, the Y dimension goes from the back to the front. Lastly, the Z dimension points straight up. By measuring these three numbers, we can find the position of any marble. This is just like the addresses in a city, which uses the street numbers, avenue numbers, and floor numbers. We thus define the meter as our unit of distance in space.

The Meter is defined as follows: Walking in a straight line from the equator to the north pole is by definition 10 million meters. One centimeter is 0.01 meters. An inch is 2.54 centimeters. A foot is 12 inches. A yard is 3 feet. A mile is 1,760 yards. Similarly, a kilometer is one thousand meters, and a millimeter is a thousandth of a meter.

The amount of matter in a marble measures its resistance to a change in its motion. Marbles like everything else are made of fundamental indivisible particles, called atoms. These atoms behave just like tiny identical marbles, since they too, like everything else, maintain their state of motion. This means that the amount of matter in a marble maintaining its motion, is then just the total number of atoms in that marble, which are all maintaining their motions. Similarly for two or more marbles, the total amount of matter is just the total number of atoms in all of our marbles. We call the amount of matter inherited to an object, as its mass, and is measured in grams.

The Gram is defined as follows: By definition, a cubic centimeter of water has a mass of a gram. One ounce is 28.349523125 grams. A pound is 16 ounces. A ton is 2 thousand pounds. Similarly, a kilogram is one thousand grams, and a milligram is a thousandth of a gram.

As it turns out, it doesn't matter what units of measurement we use to describe the marbles, the result is always the same. As an example, we could define a totally new system of units with hours, miles, and pounds, instead of seconds, meters, and grams. Even though the numbers will be different, the physics of our two marbles will be

exactly identical. This is true, even in different setups. We can therefore get rid of our units of measurement, and claim that the physics of the marbles will work regardless. With the physics of our two marbles, it seems like our lives are also a linear mechanical progression, because like the marbles, we too are made of atoms. We then conclude that our clockwork universe is made up of atoms meaninglessly drifting in the void, and is completely deterministic. This is sadly just the way it is. Or is it?

Section 1.3: Powers of Ten

I came across a new idea, one that puts humanity into perspective: Powers of Ten! Upon seeing the video by Charles and Ray Eames, I was quite amazed. I existed on level 0. Cities, forests, and oceans on level 4. Planet Earth was on level 7. The sun and solar system on level 13. The nearest stars on level 17. The milky way galaxy was on level 21. And the cosmic web was on level 24. The universe itself went out to level 26. We are nothing in comparison to the grand sceme of things. That is why I would like to share with you the true scale of reality, as a visualization tool.

Here are three tables showing the powers of ten in time, space, and matter. I arbitrarily chose 100 microseconds, 2.54 microns, and 100 grams, as the reference zero point. This will be of importance later when we discuss the scale compressed universe.

However, if one actually built such a universe, then the next rungs in the scale ladder of the actual universe would coorespond to the following entries here:

Time:	Space:	Matter:
100^0	100^0	100^0
100^5	100^3	100^-1
100^5	100^5	100^5
100^5	100^5	100^-5

Since the powers of ten are square roots of the powers of a hundred, we double the exponents. Lets create a conversion chart showing this table yet again. Imagine a particle with a mass of 100 grams, that travels 2.54 microns, in a 100 microseconds. This would be our zero point we chose. Because anything^0 is 1. And one of "our units" is the same as "our units".

Now for the next three entires of 100^5 in time, we have an elapsed time of 1 week, 4 days, 13 hours, 46 minutes, 40 seconds. Truely slow compared to the fast pace of 100 microseconds. With our table going out to 100^30, or 10^60, or approximately 2^200, we see why such a scale compressed universe is helpful. But such a universe is for later in the book.

Now we have two distances that are traversed in this elasped time. The row in space saying +10, 15.7 miles, 437 feet, 4 inches, and the row in space saying +6, 8 feet, 4 inches. For the shorter distance of +6, we have a particle mass of -2, which is 1 gram. For the longer distance of +10, we have two possible particle masses of +10, and -10. A mass of +10 is 1 megaton, whereas the mass of -10 is 10 nanograms.

For all of these examples, these three tables give you a taste of what is to come. We only have 10 powers of 10 for both time, and space. And although 10 billion is a very big number, the universe looks almost infinite in comparison. But thats they key word, "almost". Such a number like 10^60 looks like infinity but is still finite. So although we stop at the first rung, know that these tables are incomplete, centered on daily human experience. Here are the tables:

Powers of Ten: Time

0 100 microseconds
+1 1 millisecond
+2 10 milliseconds
+3 100 milliseconds
+4 1 second
+5 10 seconds
+6 1 minute, 40 seconds
+7 16 minutes, 40 seconds
+8 2 hours, 46 minutes, 40 seconds
+9 1 day, 3 hours, 46 minutes, 40 seconds
+10 1 week, 4 days, 13 hours, 46 minutes, 40 seconds

Powers of Ten: Space

0 2.54 microns
+1 25.4 microns
+2 254 microns
+3 2.54 millimeters
+4 1 inch
+5 10 inches
+6 8 feet, 4 inches
+7 83 feet, 4 inches
+8 0.1 miles, 305 feet, 4 inches
+9 1.5 miles, 413 feet, 4 inches
+10 15.7 miles, 437 feet, 4 inches

Powers of Ten: Matter

-10 10 nanograms
-9 100 nanograms
-8 1 microgram
-7 10 micrograms
-6 100 micrograms
-5 1 milligram
-4 10 milligrams
-3 100 milligrams
-2 1 gram

-1 10 grams
0 100 grams
+1 1 kilogram
+2 10 kilograms
+3 100 kilograms
+4 1 ton
+5 10 tons
+6 100 tons
+7 1 kiloton
+8 10 kilotons
+9 100 kilotons
+10 1 megaton

In fact, the universe does actually have a scale boundary. In fact there are three of them. I will name them C,G,H after the universal constants of physics. For now, don't get bogged down in the physics, these are place-holders for later. For now, they are less of a statement saying "This new pillar of physics has this constant" and more of, "Here is a scale boundary of a domain of study in our universe". Lets review them next…

Section 1.4: Universal Constants of Physics

To summarize each pillar of physics, we need to know why C,G,H are the pillars used and not some other. Remember that the 3 limiting factors of our universe are that define the boundaries of measurement scales. Each of these boundaries is actually a universal constant of physics. And because some scales are ruled out, and not others, means they have units. Here is a table that gives the pillar, the physics required for that pillar to be true, the constants I use in my equations, and the units of measurement for that constant as shown. Here is the table:

#:	physics:	constant:	units:
C	special relativity	1/c	s/m
G	universal gravitation	2*G	m*m*m/(g*s*s)
H	quantum mechanics	h/(2*pi)	m*m*g/s

Dont worry, I will cover the constants c,G,h, and their values in terms of seconds "s", meters "m", and grams "g", in a later chapter. When you get to the later chapters, you can use it as reference, and it will show you why all of this was necessary. Now that we know that, lets move on to the scale compressed universe. A mental tool of this is extremely helpful in understanding the vastness of time, space, and matter.

Section 1.5: A Scale Compressed Universe

Imagine a toy scale universe where all of the extreme scales are not so extreme. All of the things outside of our day to day experience will become obvious. Using meters,

grams, and seconds, here is an example. Imagine a real life scenario that evolves over 1 minute and 40 seconds. The toy scale version will have it take place in 2 seconds. Conversely, if a buzzer sound vibrates with 100 cycles every second in real life, the toy scale version will vibrate with 2 cycles per second. Instead of 1 centimeter in reality we have 50 centimeters in the toy scale. Instead of 100 meters, we have 2 meters. Instead of 100 grams, we would have 2 grams, and instead of 0.01 grams, we'd have 0.5 grams.

The rule I am using is simple. To any unit of measurement, even composite units containing many powers of meters, grams, and seconds, we always scale compress powers of a hundred to powers of two. Here is a table that shows how this happens:

#	2^#	100^#
-3	0.125	0.000001
-2	0.25	0.0001
-1	0.5	0.01
0	1	1
1	2	100
2	4	10,000
3	8	1,000,000

The # column is the exponent that tells us which domain of reality we are in, macro, or micro. The powers of two column denoted by 2^#, is the result of a measurement done in the toy scale universe. Whereas, the 100^# are the powers of one hundred, which explode toward infinity, or implode close to zero. We can see that a unit of measurement with a value of 0.5 or a half, in the toy scale universe that is scale compressed, maps exactly onto 0.01 or one percent in real life. This is what allows one to easily grasp the relative scales of things in nature.

Since this expands our horizons on to these extreme scales we mentioned earlier in the book, it becomes interesting to find out what kinds of physics will come into play at such scales? So far, we don't know what happens at such extremes. Could it be that no new physics shows up, and everything about our two marbles scales perfectly? This would mean that every scale is perfect and we do indeed live in a mechanical linear progression, as every measurement will be exact.

The other possibility is more exciting, and devastating to our common sense worldview. If new physics does appear, what would it be like to experience it first hand in the toy scale universe? As it turns out both our universe and the toy scale universe both share this feature of introducing new physics at it's perimeter. And, by boundary, I don't simply mean that the universe is finite, as space, time, and matter, could go on forever. This is thinking linearly, and additively. As far we know there is no end in this sense.

However, the boundary I am talking about here is that of scale. The universe likely is infinite, but our view of it is finite. By thinking in a logarithmic fashion, we hit a brick wall, where the new physics sets a scale that one cannot pass. It is a great

excersize to think of it, not as a scale factor being limited, but as the scale of the limiting factor itself. Not because our senses are bad, but because new physics does appear that constrains our view no matter how good our vision of it is.

The toy scale universe is a perfect example of this. It brings these limits very close to our everyday scale of things. Our view of scales thus expands way out to both the infinite and infinitesimal, all at once without causing confusion about big or small numbers. It is a great start at learning new physics, because at these scales everything becomes intuitive.

Like the last table, we are using both powers of 2 and powers of 100. But here we only list the exponents, so be careful. We can read off the first few rows of both toy scale and real scale. There is the scale where all limits of new physics interact, we call these new physics conepts: C, G, and H. Don't worry what they are yet, but if you feel up to the task, you might guess them based on which scales they limit, and by how much.

Take the time scale of 5 for example. If this were in seconds after the new year, the toy scale would be just 2^5 or 32 seconds since the common era began. For the real scale universe, this would correspond to year 316 AD, the date would be November 20th, which is on the 3rd day of the week, and the time of day: 5:46 PM and 40 seconds. Guess the elapsed time? It would be 100^5 seconds, otherwise put 10 billion seconds, or in science terms 10 Giga-seconds.

So clearly the toy scale universe makes a scale compressed universe intuitive and brings complex physics into the everyday. The table also includes the physics required to understand the happenings at any scale of the universe considered. A rule of thumb is that if your close to a limiting factor, you'd better include the physics cooresponding to every limit you are close in scale to. Here is the rest of the table:

time scale	space scale	matter scale	
2^#	2^#	2^#	CGH physics for toy scale
100^#	100^#	100^#	real scale physics required
0	0	0	CGH
5	3	-1	GH
5	5	5	CG
5	5	-5	CH
10	6	-2	GH
10	8	4	G
10	8	-6	H
10	10	10	CG
10	10	0	C
10	10	-10	CH
15	9	-3	GH
15	11	3	G

15	11	-7	H
15	13	9	G
15	13	-1	#
15	13	-11	H
15	15	15	CG
15	15	5	C
15	15	-5	C
15	15	-15	CH
20	12	-4	GH
20	14	2	G
20	14	-8	H
20	16	8	G
20	16	-2	#
20	16	-12	H
20	18	14	G
20	18	4	#
20	18	-6	#
20	18	-16	H
20	20	20	CG
20	20	10	C
20	20	0	C
20	20	-10	C
20	20	-20	CH
25	15	-5	GH
25	17	1	G
25	17	-9	H
25	19	7	G
25	19	-3	#
25	19	-13	H
25	21	13	G
25	21	3	#
25	21	-7	#
25	21	-17	H
25	23	19	G
25	23	9	#
25	23	-1	#
25	23	-11	#
25	23	-21	H
25	25	25	CG
25	25	15	C

25	25	5	C
25	25	-5	C
25	25	-15	C
25	25	-25	CH
30	18	-6	GH
30	20	0	G
30	20	-10	H
30	22	6	G
30	22	-4	#
30	22	-14	H
30	24	12	G
30	24	2	#
30	24	-8	#
30	24	-18	H
30	26	18	G
30	26	8	#
30	26	-2	#
30	26	-12	#
30	26	-22	H
30	28	24	G
30	28	14	#
30	28	4	#
30	28	-6	#
30	28	-16	#
30	28	-26	H
30	30	30	CG
30	30	20	C
30	30	10	C
30	30	0	C
30	30	-10	C
30	30	-20	C
30	30	-30	CH

So to conclude, our real scale universe ends at 100^30, or the present day, because since the big bang, only so much could have happened since this primordial expanding fireball. As of now, this is 13.8 billion years and counting. As an interesting take away note: Given that the origin of the whole universe is this past time horizon, frozen in time some 13.8 billion years ago, the toy scale universe is actually just our universe, viewed at a time right after the beginning. Its the fact that so much has happened since this initial moment, that makes the real scale universe such an interesting place to be in.

Chapter 2: The Three Pillars of Modern Physics

This was the physics of the industrial revolution, written in 1687 by the greatest scientist who ever lived. Isaac Newton. However, by the turn of the 20th century, experiments began to show that Newtonian mechanics is not always right, and had to be modified to agree with these radically new groundbreaking experiments. There are three main sets of new phenomena, that were incorporated into the three pillars of modern science. The three pillars are special relativity, universal gravitation, and quantum mechanics. The fate of humanity rests on the unification of physical law.

To summarize this epic conclusion, each of these pillars of physics, has radically altered our understanding of our two marbles. Our common sense vision of life as a mechanical linear progression, is just all wrong. Every pillar of physics, introduces a new universal constant. The associated physics reduces to our common sense, by making a series of approximations. So even though our common sense worldview works well in the everyday, in the grand scheme of things, our universe is stranger than fiction.

Section 2.1: Special Relativity

The dimensions of space and time, arrange themselves such that the speed of light is the same, from all moving perspectives. This means that among many new phenomena, fast moving clocks run slow, fast moving rulers contract along their direction of motion, high speed motion breaks the simultaneous nature of time, and most importantly nothing moves faster than the speed of light. The fact that the speed of light moves at the same fixed speed no matter how its source is moving, means that the dimension of time rotates in an opposite way, than the three other dimensions of space rotate.

In the theory of Special Relativity, there is a Universal Constant. The constant of Relativity has a value of 2.99792458 * 10^8 m/s, in our system of units. We recognize this as the speed of light. This means that the speed of light is very fast. To get an idea on how fast this is, imagine a foot long segment of fiber optic cable. The latency of your internet connection would be a nanosecond. A nanosecond is a billionth of a second. Therefore, we can continue to say that, time runs at the same universal rate for both marbles. Despite the fact that these relativistic effects can be ignored in the everyday regime, we still live in a universe dictated by the physics of: special relativity. The fact that special relativity works at all, shows us just how weird our universe really is. Special Relativity was put together by Albert Einstein in 1905, in order to describe the strange behavior of the speed of light, along with four other papers that shake physics down to its foundations.

Special relativity saturates at the speed of light, as anything faster is impossible, due to the notion that time would be a square root of a negative number. But, at slower speeds one recovers lorentz covariant motion with the speed of light being a constant for all motion, and slowing down much slower as compared to light speed one recovers the common sense galilean covariant motion with absolute time and absolute 3D euclidean space.

Here is the equation for many particles in special relativity:

$$\frac{d^2[t_N, x_N, y_N, z_N]}{c^2 dt_N^2 - dx_N^2 - dy_N^2 - dz_N^2} = 0$$

```
{ {frac{d^2 [t_N,x_N,y_N,z_N]}{c^2 dt_N^2 - dx_N^2 - dy_N^2 - dz_N^2}}=0 }
```

As we can see, its actually 4N separate equations, where N is the number of particles. So for each particle, one has four equations, one per particle for the coordinate time evolution per its space-time interval, and three equations per particle for the spacial evolution of the x,y,z coordinates per its space-time interval. With the space-time interval given in the denominator in which we are differentiating the coordinates against, we first note that the proper time per each particle depends on how each particle is moving throughout spacetime.

The second order derivative tells us that the "acceleration" or rate of change of rate of change of the space-time coordinates is zero. This means that the velocity given by the rate of change of these coordinates is a constant, and this constant depends on both how the particle is moving, and how it is measured. This means that every particle traverses space-time in a perfectly constant velocity, straight line.

When the velocity of any particle through space is large, the velocity of that particle through time is also large. But when the spacial velocity is very small compared to speed "c", the temporal velocity almost settles to a common rate. In the non-relativistic limit where c tends to infinity, the spacial velocity becomes linear and additive, and time ticks at a universal rate. Thus the speed c, is a ratio between the spacial velocity and a temporal velocity, or a slope in spacetime.

We also note that this slope c is constant for all particles. From any given moving perspective, the past light-cone, which is all events in space-time that reach the current particle at speed c, is compressed and sped up in front of the particle's direction of motion, yet is proportionally expanded and slowed down behind the particle's direction of motion. This kind of transformation is known as a lorentz boost, and is one of the ways to keep the speed c constant in space-time.

In fact, its not just the speed c that remains constant, but the entire space-time interval. This is why the proper time of any particle in special relativity remains a "universal" quantity we call an invariant, or in other words, its the time any particle by itself experiences, and this time, the proper time, can be computed by the space-time interval from any other particle's point of view. This means that not only lorentz boosts,

but also spacial rotations, and also space-time translations keep this proper time quantity invariant for all observers.

When any particle "sees" another particle, any influence must traverse at a speed or slope in spacetime less than speed c. We call such an influence a light cone. When we mentioned the 13.8 billion year old universe. The light cone from planet earth, our point of view, only would extend 13.8 billion light years away, this is the reason for the 26 power of ten limit in the YouTube video by Charles and Ray Eames, as anything beyond is unknown.

The same is true on the microscopic scale, when we said that the speed of light c is one foot per nano-second, this means that the computer and you reading this, can only know what you are seeing only after that time passes. For 1 foot, its 1 nanosecond or 1 billionth of a second. Across the continent is over 1 hundreth of a second, and the Apollo and Artemis moon missions had at their furthest point about only two seconds of latency, which for those that remember, is a noticable wait. For even farther distances the lookback time or retardation is even longer.

Section 2.2: Universal Gravitation

All particles attract all other particles, with an inverse square force field, in three dimensional space. This is why all objects fall to the ground at the same rate, why the planets move around the sun in ellipses with the sun at one focus and the other focus empty, and why a line drawn from the sun to a planet sweeps out equal areas in equal time intervals, and also explains why the semi major axis of a planet's orbit cubed is proportional to the orbital period squared, along with many other gravitational effects. It is the gravitational acceleration of space itself, sourced by a particle, that spreads out according to an inverse square law in 3D space. This is what keeps your feet on the ground, and the earth orbiting the sun.

In the theory of Universal Gravitation, there is a Universal Constant. The constant of Gravity has the value of 6.67408 / 10^14 m^3/g/s^2, in our system of units. This means that the strength of gravity is very weak. To get an idea on how weak gravity is, imagine two cars in deep space, each weighing a ton, parked right next to one another a yard apart. It will still take many hours for the two cars to stick together. Therefore, we can continue to say that, both marbles each maintain their motion, as they both independently travel in straight lines with a constant speed. Despite the fact that these gravitational effects can be ignored in the everyday regime, we still live in a universe dictated by the physics of: universal gravitation. The fact that universal gravitation works at all, shows us just how weird our universe really is. Universal Gravitation was discovered by Newton himself while trying to unify the motion of heaven and earth, and was published in his book, called the Mathematical Principles of Natural Philosophy.

Universal gravitation saturates for circular orbits, as anything in circular motion, in an effective radial potential will have a minimum of energy somewhere, and in this case

it is that of circular orbits. But by boosting at the closest approach, one can increase the eccentricity of the orbit, recovering Kepler's laws of orbital motion, and increasing the eccentricity of the orbit much more, the orbit breaks open into a flyby orbit that essentially passes the gravitating mass in a straight line.

Here is the equation for many particles in universal gravitation:

$$\frac{d^2[x_N, y_N, z_N]}{dt^2} + \sum_{n \neq N} \frac{G M_n [x_N - x_n, y_N - y_n, z_N - z_n]}{\left(\sqrt{(x_N - x_n)^2 + (y_N - y_n)^2 + (z_N - z_n)^2}\right)^3} = 0$$

```
{ {frac{d^2 [x_N,y_N,z_N]}{dt^2}} + sum from n <> N  {frac{G M_n [x_N-x_n,y_N-
y_n,z_N-z_n]}{left(sqrt{(x_N-x_n)^2+(y_N-y_n)^2+(z_N-z_n)^2}right)^3}}= 0 }
```

As we can see, we have 3N separate equations, not 4N like last time. Since we are talking about universal gravity and are in the non-relativistic limit, the proper time, retarded time, and coordinate times, all coincide exactly. This makes the 4th equation trivial and it vanishes identically. The other three equations per particle tell how much the spacial velocity changes in the x,y,z directions as time passes. Since this three dimensional acceleration is now non-zero, lets now see what happens to the particles in universal gravitation.

For all pairs of non-identical particles, there is an acceleration that depends on the mass of the other particle, where the other particle is, and how far away it is. In fact, the inverse square law applies, which means that the farther away any two pairs of particles are, the weaker gravity becomes. This is what I meant by "an inverse square law in three dimensional space". In other words, the rate that the velocity of a particle changes depends on the other particles around it.

Kepler's laws are reproduced for any pair of particles in isolation. But for many particles like we have here, the result can be quite chaotic. Gravity is not just a simple pull, as it can fling entire objects deep into space. The result is a cosmic web, full of intricate filaments and sparse voids. Zooming in, clusters of particles are held together by their own self-gravity.

The constant "G", tells us how strong this self gravity is, and sets the scale for gravitational interacting systems, like our solar system, our galaxy, and our universe. Since special relativity exists, means that ultimately this equation must also give way and unify when speeds get high, but here they are exactly described by newton's laws of motion and gravitation shown here.

Section 2.3: Quantum Mechanics

Every wave can be thought of as a stream of particles. The total kinetic energy of such a particle is directly proportional to its corresponding probability wave frequency. Moreover, waves with shorter wavelengths correspond to identical particles with more momentum, that is to say, the product of mass and velocity. This leads to the notion that objects can be in more than two places at once, how particles and waves are actually the same thing, how these quantum waves have discrete harmonics like the specific notes on

a musical intrument, and the weird notion that measuring one particle can reveal corelations in the properties of all other particles of the same type.

In the theory of Quantum Mechanics, there is a Universal Constant. The constant of Quantum has a value of 6.62607015 / 10^31 m^2*g/s, in our system of units. This means that the amount of quantum uncertainty is very certain. To get an idea of how certain quantum physics is, imagine a grain of dust, weighing less than a microgram, sitting on your desk at school. After 30 years, your kid, now sitting at your desk at school, notices that the same exact dust grain has moved by a millionth of an inch. Therefore, we can continue to say that, the two marbles both have definite positions as well as definite velocities. Despite the fact that these quantum effects can be ignored in the everyday regime, we still live in a universe dictated by the physics of: quantum mechanics. The fact that quantum mechanics works at all, shows us just how weird our universe really is. Quantum Mechanics was constructed by a band of nuclear physicists in order to explain the color spectrum of a fluorescent light source.

Quantum mechanics saturates for a bell curve wave-packet, with the uncertainty between momentum and position at a minimum. This is due to the fourier or harmonic series giving a bell curve probability distribution for both spacial location, and for spacial frequency. But for more spread out, decoherent wavefunctions, the uncertainty between position and momentum widens, and this recovers the wavefunction evolution of the shrodinger equation for a free quantum particle, and by increasing the wavefunction randomess even more, when zoomed out, one has recovered the motion of a free point particle.

Here is the equation for many particles in quantum mechanics:

$$\frac{d\Psi}{dt} - \sum_N \frac{\hbar\sqrt{-1}}{2M_N}\left(\frac{d^2}{dx_N^2}+\frac{d^2}{dy_N^2}+\frac{d^2}{dz_N^2}\right)\Psi = 0$$

```
{ {frac{d %PSI}{dt}} - sum from N {frac{hbar{sqrt{-1}}}{2M_N}}
left({ {{frac{d^2}{dx_N^2}}+{frac{d^2}{dy_N^2}}+{frac{d^2}{dz_N^2}}}}right)
%PSI = 0 }
```

As we can strangely see, there is only one equation, that acts on a single "thing" we call the wavefunction. Denoted by the capital greek letter PSI, the wavefunction depends on all the positions of all particles at a given time t. The square root of negative one is doing a special trick here in which the wavefunction "rotates" around as time passes, and the frequency of this so called rotation depends on the energy of the particles in question. If you know about complex numbers denoted by a plus b times the imaginary unit, where the imaginary unit is the square root of negative one, then you should be familiar with this equation.

The mass of any particle, plugs in to both the particle's non-relatistic kinetic energy which is half the dot product between its velocity and linear momentum. The linear momentum is just the particles mass times its velocity. So, this means that a particle that "moves" faster through space, will have a branch or in other words a set of 3 spacial coordinates of the wavefunction, that "rotates" more in a given region of space.

In other words, what we call a particle is nothing more than a ripple with a definite frequency and wave-length. The constant "h", sets the scale of these so called plane waves. When the mass of any particle gets large, the scale of these waves gets microscopic for that branch of the wavefunction. The magnitude of the wavefunction squared integrated across all space tells you the probability of finding a particle. And by projecting such a measurement across all 3N dimensions of the wavefunction, one can in principle calculate the probability of finding said particles at their said locations.

What makes this interesting is how the waves interact after such as measurement, as when a particle is observed, it modifies the whole wavefunction it sits in, affecting the motion of all the other entangled particles. However, this quantum mechanical equation lacks both special relativity as we have a single universal time dimension, and universal gravitation, because the wave-like "packets" of probable wavefunction densities flow freely from one location to another. What one needs is to unify quantum mechanics with both special relativity and with universal gravitation. This requires redefining our assumtions and letting go of some long held assumptions we made for the other two frameworks for many particles. For this, lets begin with a single particle, before we move on to many particles like we had in this chapter.

Chapter 3: A Theoretical "CGH" Physics Cube

Here we have the mathematics that unifies special relativity, universal gravitation, and quantum mechanics, for a single particle viewed from the origin. You are welcome to skip it, as the math involves derivatives and many high level math ideas. However, if you want to get a grasp on what the math says about each physics domain, or if like me, you are planning on building a physics game engine in python, then stick with me here.

I have the definitions of all the equations intact, so if you see something unusual, just rewind a bit to the earlier definition. Every equation should be defined as a partial differential equation involving a left hand side that involves the variables x, y, z for 3D space, t for time, and M for the particle mass. The right hand side is always zero for the defining equations.

Each equation describes the physics of a single point particle. So there is only one particle, the observer is identical to the particle, except that it is pinned down at rest to the origin for all times. Therefore one can say that all 8 equations answer the question, "If I were a particle, what would I look like, when seen from my very own outsider's perspective?". Since this is a physics problem, it is a perspective shift for understanding a single particle.

Each of the 8 equations, toggles the physics of C,G,H, binary wise. In the scale compressed universe, what I call a "toy scale" universe, I mentioned which physics applies at which scale. As we can see, each pillar of physics is either required or absent. This gives 2^3 unique domains, for a total of 8 defining equations.

Now for the fun part, as we are about to reveal what the 8 equations are. If you are scared of the complexity, don't worry, they might not make sense at first, but as soon as I explain them, you will get more familiar with them as we go. If it's too opaque, you are welcome to skip forward. Now here we go…

Section 3.1: A Galilean Transformed Particle

We begin with a single particle. Without any of the three pillars of physics active in such a domain, we arrive at the following equation.

$$\frac{d^2[x,y,z]}{dt^2}=0$$

```
{ {frac{d^2 [x,y,z]}{dt^2}}=0 }
```

It says that the acceleration of a particle in 3D space is zero. Given this, the particle's velocity can take on any value, but stays at that velocity moving in a perfectly straight line, with a constant speed. Therefore there is no gravity physics in the equation. The particle has a definite position and velocity, as one can calculate it exactly

from any initial condition. This means that there is no quantum physics in the equation either. With an absolute time running in the background, there is no relativity physics in the equation, which means we lack all three pillars of physics. Lets turn the binary crank and see what comes out next out of our CGH physics machine.

Section 3.2: A Lorentz Transformed Particle

This equation looks very similar to our special relativity equation. In fact, it describes only one particle obeying special relativity. The same physics applies here, so I will just present the equation and move on from there. Here it is.

$$\frac{d^2[t,x,y,z]}{c^2 dt^2 - dx^2 - dy^2 - dz^2} = 0$$

```
{ {frac{d^2 [t,x,y,z]}{c^2 dt^2 - dx^2 - dy^2 - dz^2}=0 }
```

The single particle moves in the reference frame of an observer which is a particle that does not obey any CGH physics, but is sitting at the origin for all times. The coordinates drawn from such an observer give us the t,x,y,z grid in which our special relativistic particle is moving through. Next we turn the binary crank some more, and see what comes out next.

Section 3.3: Kepler's Orbit Problem

Ahh okay, the kepler's orbit problem came out. This equation like the previous is a single particle version of the physics. Not of special relativity, but of universal gravitation. Here it is.

$$\frac{d^2[x,y,z]}{dt^2} + \frac{GM[x,y,z]}{\sqrt{x^2+y^2+z^2}^3} = 0$$

```
{ {frac{d^2 [x,y,z]}{dt^2}} +{frac{GM[x,y,z]}{{sqrt{x^2+y^2+z^2}}^3}} =0 }
```

Now, we have the same setup of an observer at the origin, and the gravitating particle going around it. Wait, this seems backwards, the Earth orbits the Sun, because the sun has much more mass. Right, but one must realize that we explicitly chose the test particle to be at the origin. Since such a rebasing leaves the physics unchanged, one can indeed say that the orbit is correct. We continue to turn the crank. Next up is one of the most fundamental ideas I ever had. Black Holes!

Section 3.4: Schwarzschild Metric Geodesics

To unify special relativity with universal gravitation, imagine two gravitating marbles. Placed at rest an infinite distance apart, the two marbles will gradually begin to accelerate toward one another. The speed needed to break the fall, is known as the escape velocity, and is also the speed at which space itself carries the marbles together. However, we know that gravity spreads out with distance, meaning that the escape velocity would exceed light speed at a close enough distance. In a sense, space itself is falling inwards, faster than light can. And since nothing can propagate faster than light,

everything inside this distance is trapped forever. This is the event horizon of a black hole!

Going back to our two marbles, we know that they are actually two black holes. The mass of each black hole, is simply the size of its own event horizon. The year is 1915. Albert Einstein completes his General Theory of Relativity. Weeks later, Carl Schwarzschild found the spacetime geometry around a simple black hole. The Schwarzschild spacetime reduces to special relativity and universal gravitation, in their respective domains. However, black hole orbits do things not seen before in science. Time slows down, light bends, orbits precess around. Closer in, gravity increases like a rip tide, rendering circular orbits unstable. Even closer, and it is possible for light to orbit temporarily. Crazy stuff.

For fun, lets dive into a Schwarzschild black hole. Far away from the hole, its gravity seems normal, since the paths of objects around it, the orbits, behave as we would expect. The slowest orbits are the elliptical orbits, like the earth around the sun for example. Faster orbits, like that of a comet passing by a planet, are known as fly-by orbits. The fastest orbits are the straight lines that light takes to reach your eyes. Closer in, say within a few dozen black hole radii away, the gravity is so powerful that light rays will noticeably bend, just like the flyby orbits we mentioned. On the other hand, the slower elliptical orbits are also acting very strangely. They undergo a phenomenon known as precession, which is to say that the closest point of the orbit, the perihelion, shifts in the same direction of the orbit itself, turning a simple ellipse into a daisy pattern. Then, at three black hole radii from the black hole, circular orbits become unstable. This means that a small push will either send us flying into the hole, or off into deep space. At this distance, orbits zoom and whirl around the hole. The orbital precession and time dilation effects are clearly noticeable. Closer in than three radii from the hole, faster and faster elliptical orbits become unstable, until we arrive at two black hole radii from the black hole, at which flyby orbits start being unstable too. The only orbits possible are the highly bent light rays, which also follow flyby orbits. Then, at a special distance of one and a half radii away from the black hole, light can orbit the black hole. However this orbit is unstable, and the light rays either fly off to infinity, or plunge into the black hole. Even closer than this, and orbits do not exist, since the orbital speed needed would be faster than light. Just above the event horizon, it is still possible to escape the gravity of the black hole. However, as we approach the horizon at one black hole radius, the chance to fly away gets slimmer and slimmer. Time slows to a halt, when seen from the outside. Then as you cross the invisible spherical boundary, you are doomed to hit the central singularity, where everything you know comes to an end. Seen from outside however, nobody even sees you cross the event horizon at all, as your image fades away on the black hole surface.

To describe such orbits, one needs to choose a coordinate system for such Schwarzschild or non-rotating black holes. There are a few problems with most

coordinate systems that are useless for our purposes. The first is the problem of coordinate singularities. In other words, there are places or sometimes entire regions of space-time cut off from our view. The event horizon is one good example. If one is stuck in spherical coordinates, the poles along the Z axis are another example. This is why we have been using cartesian coordinates, all along is to remove the singularity at the poles.

Note the we have an observer that orbits the particle. Like the Kepler orbits, one can rebase our observer back to the origin using a galilean transform, making the apparent mass of the particle appear to be located at the origin. In the relativistic case, we can also rebase the observer to the origin, but this time using a lorentz transform. Like before, the mass of the particle seems to be located at the origin. So for all intents and purposes, the observer and particle seem to switch roles. Instead of a test particle going around a black hole, we have the opposite: A black hole going around the test particle.

For the second issue, we note that for such a symmetry between a black hole and test particle to occur, in order to rebase the observer, there needs to be a way to have the coordinates to "align" with the "image" the black hole produces to itself. Since we are dealing with a relativistic spacetime, such rebasing must respect lorentz symmetry, and therefore the past light cone, which is invariant under such a symmetry, must be present globally, in order for the black hole to sit at the origin as its own observer. By establishing that the Eddington-Finkelstein coordinates work for this purpose, we transform the Eddington coordinates into cartesian Kerr-Schild form, which eliminates all coordinate singularities including the event horizon.

One might wonder why we are using full blown general relativity here, and not just a relativistic version of newtonian gravity. I mean why all the complexity? The schwarzschild spacetime is a solution to the vacuum Einstein field equations, which are a nonlinear set of relatistic wave equations, with the mass and spin invariants being massless and with spin equal to 2. Unlike the massive, spin 0 case we are advocating for here for the particle, the gravitational field seems way too intricate.

The reason stems from a symmetry we will define later, equating the global spacetime symmetries, like that of lorentz boosts, to local symmetries that define the force field, that might have the same symmetries as the space-time the force field it inhabits. Einstein's huge breakthrough was that the gravitational field is spacetime, not a field on spacetime but spacetime itself. For this to work properly, we need to match the local and global symmetries. We do this in a later chapter, but for now, just know that his equations do indeed have the same symmetries for space-time, which is our case is a massless spin-2 field. So don't worry if the space-time seems more complicated than needs be, it has nothing to do with the particle content, as one can couple a gravitating source to any field, be it a scalar field or a tensor field like we do here.

Moving on, we notice a fourth reason for choosing such a spacetime. We removed the singular regions for tractability, chose orthogonal light cone respecting coordinates

to rebase the observer, and noted that a full spacetime geometry is needed for consistency reasons. Now, with such Eddington-Cartesian coordinates for the Schwarzschild spacetime geometry, we can see that unlike other coordinates that distort our view, the same reason we chose light cone adapted coordinates, allows for an arbitrary amount of mass for the black hole.

This is useful because mass is just a parameter, like time and space. If one stacked up every black hole in this fifth dimesion, the black holes of other coordinate systems would not agree with each-other due to the distorted coordinates. But here in Eddington-cartesian coordinates, the light-cone symmetry allows for a linear perturbation of each Minkowski space-time slice. And because its linear, superposing solutions holds so far, at least for the origin.

Now that we made our choice, its time for the math. Here is the spacetime 4-vector for relative positions and elapsed times. This equation is the definition of a spacetime vector. It groups time t, with the spacial dimensions, x,y,z into a four dimensional vector, sometimes called a 4-vector. The greek "mu" is an index, not an exponent. Here, these indeces label, which spacetime dimension we are using for an equation. So equation mu equals t, gives the equation to be solved for time, the next for mu equals x, for the x equation. And so on, for all 4 equations. Its just a compact way of writing.

$$dx^{\mu} = [dt, dx, dy, dz]$$

```
dx^%mu = [ dt,dx,dy,dz ]
```

Then we define the radius, as the norm of the spacial coordinates. To make this work, we define the radius r, that tells you how far away the particle is from its origin. This is the simple pythagorean theorem in three dimensions. From it, the orbit equation can be used in full.

$$r = \sqrt{x^2 + y^2 + z^2}$$

```
r = sqrt{ x^2 + y^2 + z^2 }
```

Here is the gravitational potential, written in dimensionless units. It is the magnitude of the pertubation and is the sole reason for gravitational time dilation, even at rest where in special relativity time flows as usual. It is this discrepancy in the flow of time that causes gravity in the first place. This is the gravitational potential, or degree of distortion, near the black hole.

$$H = \frac{2GM}{c^2 r}$$

```
{ H = frac{2GM}{c^2 r} }
```

And here is the lookback time vector, that points radially in from outside along the past light cone. This means that, alongside the magnitude, we have a direction. This direction points inward at the speed of light. In fact, it tells us that the gravitational field only responds at the speed of light. That is a hallmark of all theories that obey special relativity. The farther away the particle is, the more time it takes for the light-speed influence to relay information to us. We call this the lookback time, or retarded

time. In other-words, we can think of this vector as the flow of gravitational information into the origin.

$$l_\mu dx^\mu = c\,dt + \frac{x}{r}dx + \frac{y}{r}dy + \frac{z}{r}dz$$

```
{l_%mu dx^%mu = c dt + {frac{x}{r}} dx + {frac{y}{r}} dy  + {frac{z}{r}} dz}
```

Lastly, and most important, is the spacetime interval. Notice the minus sign. That is crucial, for knowing that t is a time dimension. The reason: If the interval is zero, it must remain zero for all spacetime frames, the relativity principle, which means that is the reason for replacing the absolute time, with another absolute quantity. But we know the speed of light is absolute. Not time, not space, and definitely not angles. So the minus sign keeps the light speed constant, by making sure that time and space balance out to a constant value. In this case: zero. And with this in mind, the Minkowski space-time interval given by the following expression:

$$\eta_{\mu\nu} dx^\mu dx^\nu = -c^2 dt^2 + dx^2 + dy^2 + dz^2$$

```
%eta_{%mu%nu} dx^%mu dx^%nu = -c^2 dt^2 + dx^2 + dy^2 + dz^2
```

Alongside it, we have the 4-vector gradient, and we define partial derivatives as follows. It is simply the rate of change in time for the time component, and the gradient operator for the spacial components.

$$\partial_\mu = \frac{d}{dx^\mu}$$

```
{ partial_%mu = {frac{d}{dx^%mu}} }
```

We can first define the inverse metric tensor for gradient vectors. The metric tensor has an inverse. Useful for computing the forces in spacetime due to parallel transport, we can see that the inverse is simply a sign flip of the potential, and a reversal of time for the lookback time vector. Everything works here too, thanks to the linear nature of the cartesian coordinates.

$$g^{\mu\nu} = \eta^{\mu\nu} - H\,l^\mu l^\nu$$

```
{ g^{%mu%nu} = %eta^{%mu%nu} - H l^%mu l^%nu }
```

And also define the usual metric tensor for position vectors. Like from special relativity, we have a spacetime interval. However instead of using the flat Minkowski spacetime, we use the curved Schwarzschild spacetime in cartesian-Eddington coordinates. This means that the distances and elapsed times are exactly perturbed away from the flat spacetime case.

$$g_{\mu\nu} = \eta_{\mu\nu} + H\,l_\mu l_\nu$$

```
{ g_{%mu%nu} = %eta_{%mu%nu} + H l_%mu l_%nu }
```

The space-time interval of any two events in spacetime is no longer a simple Minkowski metric, that works for any separation. No. Because the metric is now describing a curved space-time geometry and not a flat one, one can only use infinitesimal spacetime separations because, the flat Minkowski spacetime is only recovered when one zooms way in on the geometric continuum. But together, the new spacetime interval, becomes useful when one wants to know how much time has elapsed along a path. This depends on both how far away you are from the black hole, and on how

fast you are moving relative to it, amonst other things that the metric tensor allows. Here, the path in consideration is for a moving observer in such a geometry. We call such a time that you personally experience as proper time:

$$ds^2 = -g_{\mu\nu} dx^\mu dx^\nu$$

```
ds^2 = - g_{%mu%nu} dx^%mu dx^%nu
```

This allows one to take derivatives of the metric tensor to find the christoffel symbols, this is very similar to how one finds the force by taking the gradient of the newtonian potential. We do exactly that in the following equation. And then this defines the full geodesic equation for our particle in orbit around its own black hole:

$$2 g_{\mu\sigma} \frac{d^2 x^\mu}{ds^2} + \left(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu}\right) \frac{d x^\mu}{ds} \frac{d x^\nu}{ds} = 0$$

```
{2 g_{ %mu%sigma } { frac{d^2 x^%mu}{ds^2}} + left({partial_%mu g_{ %sigma%nu }
+ partial_%nu g_{ %mu%sigma } - partial_%sigma g_{%mu%nu}} right) { frac{d
x^%mu}{ds}} { frac{d x^%nu}{ds}} } = 0
```

This equation can be solved numerically on the computer, and the orbits are exactly like we talked about. Perihelion precession, light bending, time physically running slower closer in, and so on and so forth. And the coordinates we used really pays off. The event horizon penetrates one way, and doesn't blow up. One can switch from the cartesian coordinates to spherical, solve the equation's analytically using the Jabobi Elliptic functions and some proper time orbital integrals, and convert back to cartesian for an exact solution.

Whats amazing here is that the equations are fully unified here, without further assumptions, one has genuinely new phenomena, but are only seen and known about after the unification. For a consistency check, when orbits get far enough away from the origin, special relativistic orbits are recovered for fast, almost straight line motion, and universal gravitating orbits are recovered for slowly moving, almost Keplerian like conic sections. But the almost is key. Just by assuming that the "nothing" of space-time is fundamental, and some nice coordinate tricks, we ended up explaining black holes and their orbits. Fundamentally new phenomena coming at us from known physics. Amazing!!!

Section 3.5: Schrodinger's Wave Equation

We turn the crank yet another time, and arrive at the single particle version of quantum mechanics. Like the sections on special relativity and universal gravitation, we are reduced to a single particle moving about the origin. Lets go about defining the parts needed for this equation.

The following equation defines what is known as the Spacial Laplacian. This operator tells you how much a quantity changes in its surroudings, and is defined as the magnitude of the 3D spacial graident operator. Just like how the radius "r" is defined in terms of the coordinates x,y,z through the Pythagorean Theorem, the same is true of its inverse. In fact, applying this operator to a plane wave, scales the plane wave by the

square of its inverse wavelength. Shorter wavelengths get amplfied, longer waves are diminished. Here is the laplacian operator for 3D Euclidean space.

$$\Delta^2=\frac{d^2}{dx^2}+\frac{d^2}{dy^2}+\frac{d^2}{dz^2}$$

```
{%DELTA^2 = {{frac{d^2}{dx^2}}+{frac{d^2}{dy^2}}+{frac{d^2}{dz^2}}}}
```

Remember the imaginary unit, where the imaginary unit is the square root of negative one. We can square both sides to obtain its definition, shown here.

$$i^2=-1$$

```
i^2 = -1
```

And finally this gets us to the schrodinger equation itself. With these new definitions, we can write our single particle in quantum mechanics as follows.

$$i\hbar\frac{d\Psi}{dt}+\hbar^2\frac{\Delta^2\Psi}{2M}=0$$

```
{ i hbar {frac{d %PSI}{dt}} + hbar^2 {frac{%DELTA^2 %PSI}{2M}} = 0 }
```

As usual, we have a single quantum particle moving past an observer. Throughout all the equations in this chapter, only the particle obeys the physics we are using, with the observer a classical free newtonian particle without any CGH physics. This observer is always at the origin, and provides us with our coordinate system. With this in mind, lets continue to turn the crank…

Section 3.6: Klein Gordon Equation

To unify special relativity with quantum mechanics however, imagine two quantum marbles. You roughly know their distance and relative velocity after measuring them. Lets say we want to measure the separation between the two marbles, this time with a much greater precision. The uncertainty in the relative velocity of the marbles will have increased by a similar factor. By measuring the distance precisely enough, the uncertainty in relative velocity will exceed light speed. When this happens, you have discovered antimatter!

Going back to our two marbles, we know that they are actually two quantum waves. The mass of each quantum wave, is simply its own frequency. The year is 1925. Erwin Schrodinger finds a relativistic wave equation for these quantum waves. Weeks later, Klein and Gordon rediscover this relativistic wave equation, after Schrodinger abandoned it. These Klein Gordon waves reduce to special relativity and quantum mechanics, in their respective domains. However, Klein Gordon waves do things not seen before in science.

Just for fun, lets take a closer look at these Klein Gordon waves. Like all waves, Klein Gordon waves can be decomposed into a series of plane waves with definite frequencies and wavelengths. Take one of these plane waves for example. It is called a plane wave for a reason. Simply because as the wave scrolls through space, it has wave fronts that are flat two dimensional surfaces, or planes. The perpendicular distance between the parallel wavefront planes, is the wavelength. The frequency is just the

number of cycles per second. So in other words, a plane wave is the building block of any wave. Lets now boost to a moving perspective, such that we are moving with the wave. At this point, the wavelength is so long, that for our purposes, the plane wave is now a synchronized sine wave. Its like the tides going in and out of the shore, as the whole flat ocean seems to rise and fall every day, in perfect unison. As we boost to a slowly moving perspective, the frequency has increased very slightly, however the synchronized wave is now out of sync again, as another plane wave, with a very long wavelength. At such slow speeds, the frequency of the waves is fairly constant, and could be ignored. The remaining phase angle of our wave, now has a much lower frequency, now in proportion to the kinetic energy of its associated quantum marble. The inverse wavelength of our nearly synchronized plane wave on the other hand corresponds to the linear momentum of our marble. As we boost away with faster and faster velocities, the wavelengths of our plane wave get shorter and shorter, and the frequency increase becomes noticeable. As we boost much faster, ever closer to light speed, the plane wave now has a very high frequency and a very short wavelength, such that the plane wave now seems to travel at exactly the speed of light. Boosting more will only make the waves even shorter, as they continue to travel at exactly the speed of light.

To make this work, the equation, known as the klein gordon equation, describes a wavefunction in Minkowski spacetime for a relativistic quantum particle. This equation unifies special relativity with quantum mechanics. The Klein Gordon equation is the relativistic generalization of the shrodinger equation. It extends to relativity, by using the flat Minkowski metric. The solutions are plane waves like the shrodinger case, but this time being compatible with relativity, they travel at any speed, even close to light speed. Here it is:

$$\eta^{\mu\nu}\partial_\mu\partial_\nu\Psi-\frac{M^2c^2}{\hbar^2}\Psi=0$$

```
{ %eta^{%mu%nu} partial_%mu partial_%nu %PSI - {frac{M^2 c^2}{hbar^2}} %PSI = 0
}
```

As we can see here, the matrix denoted by the lowecase greek latter eta is the Minkowski metric tensor. In fact, its the inverse tensor because we are contracting with the 4-vector gradient, not with the 4-vector position. But the time-time component is opposite to the on diagonal space-space components as per usual. Since the wavefunction depends on space-time coordinates in this "relativistic" way, means that we recover the plane wave structure of the energy-momentum relations from special relativity, and this recovers the shrodinger equation in the non-relativistic limit, that we explained in detail above. Again like the previous case in which we unified two of the three pillars of physics, we do see new physics, predicted from the two pillars we are to unify.

The mass term always has the same sign as the time derivatives, and opposite sign to the three space derivatives. This means that the wave-function oscilates through time, and is damped throughout space. The reverse cannot be true because this would allow superluminal motion which breaks the light cone structure we established for

special relativity. Good thing to check the sign conventions of all terms. Next up is a doozy, but is good stepping stone that helps us gain intuition for the full unification of all three pillars.

Section 3.7: Hydrogen Atom Orbitals

To unify universal gravitation with quantum mechanics, we note that both obey a simple galilean transformation symmetry. For a single particle seeing itself at the origin, we let the observer at the origin be a non-gravitational and non-quantum object. This means that first, the observer doesn't accelerate due to gravity, so it remains completely in inertial motion. Second, the observer is also non-quantum, so it must have a definite position and definite velocity. Therefore we can perform a galilean transformation by just sliding or translating the positions in each slice or moment in time, such that the observer is always rebased back to the origin point at all times.

However the observer must see the quantum particle in its own gravitational field, so we note that a simple newtonian gravitational potential energy must be used. Since the quantum-gravitational particle is quantum, the shrodinger equation must also be used, like we did for the purely quantum case. The particle unlike the observer must be a gravitating quantum particle, whereas the observer is always without new physics. The reason why the gravitational potential is at the origin is because we are rebasing the falling observer to always be at the origin. This is just like our purely gravitational example, where the particle is like the sun, and the observer is in orbit around the gravitating particle, just like the earth orbits the sun. However, we can still do a galilean transformation and bring the observer at rest at the origin, as long as we accept that the apparent mass of the sun or particle the observer is orbiting seems to be where-ever we are observing it from, in this case the origin.

The shrodinger equation of mass M gives us the kinetic laplacian term and the hamiltonian term which is just an complex valued oscilitory term with a frequency that depends on both the kinetic and potential energy. Now that the potential energy is non zero we add in the potential energy to the hamiltonian, given to us by the newtonian gravitational potential energy with the observer at the origin with apparent gravitational mass M.

Notice the symmetry between the quantum and gravitational counterparts. Both have mass M. The potential itself is purely gravitational and non-quantum, so we can define it precisely. The shrodinger wavefunction is purely quantum without any gravitational backreaction sourcing anything. One could say its a quantum particle on its very own gravitational background. But this is really only a single quantum-gravitational particle as we have seen. By viewing it from the observer's point of view, the gravity and quantum counterparts seem to decouple.

You might be surprised to hear that the solutions are exactly identical to the hydrogen atom orbitals from chemistry. Chemistry uses nuclei and a single electron at a

time, usually a valence electron from density functional theory, to derive the orbitals. But its amazing that for the gravitational case, the exact same structure seen in hydrogen is also seen for the quantized kepler's orbits. Both have the same periodic table, both have the same ionization spectrum. The only difference is scale. A real atom is tiny. But if one turns off the electric force, so only gravity domainates, everything dies and goes dark, but the hydrogen atom remains. Due to the feeble pull of gravity, a proton mass black hole as our observed particle will have a atomic radius of about the size of the current observable universe. Whereas the spectrum would only be noticable a good 40 orders of magnitude longer than the current age of the universe.

However, for a much heavier particle, say 12 orders of magnitude below the planck mass, not 20 anymore, so its 100 million protons heavy or a trillionth of a planck mass, the gravitational hydrogen atom would be 36 orders of magnitude larger than the planck length about the size of a large tree, and take 60 orders of magntiude longer than the planck time, which is as we know the current age of the universe.

As we increase the mass further still, nearing the planck mass, the gravitational hydrogen atom, shrinks and picks up frequency. Until relatvistic effects begin to dominate the spectrum, and we won't tackle this until the next section. For now, just know that the hydrogen atom is a structure that appears whenever one has a inverse square force field of any kind, which is natural in a 3D space like ours, and that is why both electric and gravitational fields have this property. Even for massive particles, they can carry forces out to the distance that their mass permits, but closer in its also an inverse square potential. We will cover such Yukawa potentials later in the book.

For something quite new, we unify universal gravitation with quantum mechanics. Even without special relativity, it is amazing that the solutions for a single particle coorespond to the atomic orbitals of the periodic table for chemistry. This occurs because like for electromagnetism, which dominates the atom's electrons, gravity as we already know also spreads out into space. It is the quantum version of Kepler's orbits. Here is the equation for a gravitational hydrogen atom, for a particle in quantum-gravity:

$$i\hbar\frac{d\Psi}{dt}+\hbar^2\frac{\Delta^2\Psi}{2M}+\frac{GM^2}{r}\Psi=0$$

```
{ i hbar {frac{d %PSI}{dt}} + hbar^2 {frac{%DELTA^2 %PSI}{2M}}  + {frac{G M^2}
{r}} %PSI = 0 }
```

Now comes something fun. The limiting cases of the hydrogen atom orbtials! Before we get there, we must talk about the exact solutions. They are always defined from both the bound states and scattering states put together. The bound states are all polynomials in standard spherical coordinates, like Lagurere and Legendre polynomials that form the spherical and radial functions, but the scattering states use a simple hypergeometric function known as the 1F1 function. But doing this requires us to go to spherical coordinates and back. This holds true for any spherically symmetric force field. So when we go beyond this point, in the next few chapters, it will get really anoyying to just find exact solutions. Mention Newman-Janis trick and Newman-Penrose

formalism, or Kinnersley Tetrads, and I am sure the complexity will be so bad, that using numerical solutions will be better than analysis anyways. So the non-relativistic limit is one of the few examples that have exact solutions. Now for the limiting cases.

Universal gravitation is recovered in the classical or non-quantum limit. Like the shrodinger and klein-gordon examples, we superimpose many wave-function solutions together to recover a blob of probabable particle density. And as it turns out, such coherent states as we call them, appear to move in the way that Kepler's orbits predict, just a little fuzzy. By zooming way out, and looking at the multitude of states right next to the bound to scattering transition, the fuzzy-ness seems to get more precise, and this almost recovers universal gravitational orbits.

Quantum mechanics is recovered when the "particle" is moving fast enough to become unbound. This means that the extremely high frequency scattering states start to approximate the shrodinger equation without a potential. This is just like how all the other gravitational examples reduce to their non-gravitational counterparts. The same is true here. We just look at the unbound or hyperbolic orbits, and the faster the orbits get, the closer to straight line motion they become. When we go to even higher energies beyond the gravitational potential, it approximates the free quantum particle better at closer and closer distances. Now that we know this works, its time for the final unification, in which we unify all three pillars into one!

Section 3.8: The Klein Gordon Equation in the Schwarzschild Metric

And finally, we get to the apex equation. The equation that unifies all three pillars of physics into one: Special relativity, universal gravitation, quantum mechanics. All into a single equation. It's the Klein Gordon equation, but for the curved spacetime of the Schwarzschild metric. It can also be looked at as the relativistic generalization of the hydrogen atom wavefunctions. Here the solutions are found by some analytical functions like confluent Heun functions, a type of hypergeometric function, along with the spherical harmonics from before. This is what a particle is in all domains, even when the physics becomes irrelevant to use.

$$g^{\mu\nu}\partial_\mu\partial_\nu\Psi+\left(\partial_\nu g^{\mu\nu}\right)\partial_\mu\Psi-\frac{M^2c^2}{\hbar^2}\Psi=0$$

```
{ g^{%mu%nu} partial_%mu partial_%nu %PSI   } + {  left(  partial_%nu
{g^{%mu%nu} }right) partial_%mu %PSI    } - frac{M^2 c^2}{hbar^2} %PSI = 0
```

We did it! We have a single equation that describes a single particle of CGH physics with the observer at the origin as per usual. Now that we have an equation that applies to all three domains of physics, we can first take their limiting cases and compare. But we already have the other 7 equations as the limiting cases to CGH physics. So now we instead turn to the next part of physics: Spin! Like mass, spin is an intrinsic property of all matter and all forces. We will show why later on in the book, when we

develop our theory of everything. But for now, we need an equation as a jumping off point in order to develop such a theory in the first place. With the assumption that only mass and spin, are fundamental, we lay out hope that we can derive all of physics from it. So without further hesitation, lets put a spin on things…

Chapter 4: Generalizing to Spinning Particles

We know that rotating black holes are particles. We know that quantum matter waves are particles. We know that particles are characterized only by their mass and by their intrinsic spin. By putting these ideas together, we first know that a rotating black hole is by definition a quantum matter wave and vice-versa. In the last mathematical details section, we unified special relativity, universal gravitation, and quantum mechanics, all for a single particle with mass M. The last equation was the full CGH equation: The massive klein gordon equation in the schwarzschild spacetime geometry with any identical mass. We now know how to introduce spin correctly for both the GC and HC counterparts.

In this section, we build upon the CGH equation for a spinless particle, by including spin. Like mass, the spin of the black hole and the spin of the wavefunction must be the same. But unlike mass, which can take on any positive value, spin is defined to be multiples of spin half, like that of many identical electrons, for both the black hole and the wavefunction. It is the full CGH equation with spin half that we are after, and it will require much more technical machinery than we had for the spin zero or even the spin one case. Here comes the following equations for both spin one and after that, spin half.

Section 4.1: Kerr Metric Geodesics

In 1964, Roy Kerr found another solution to Einstein's Theory, one that extends the Schwarzschild solution to include a new phenomenon, spin! In effect, spacetime gets dragged along with the rotation, causing even more mayhem to our orbital path around the hole. This is known as frame dragging. There are an inner and outer event horizon, and even an inner and outer ergo sphere, within which it is possible to extract rotational energy. Moreover, a rotating black hole, spinning as fast as possible, has unstable orbits all the way out to nine black hole radii out going against the spin, yet can have stable circular orbits traveling along with the spin of the black hole, all the way down to the outer event horizon itself. Even so, the Kerr spacetime, reduces to the Schwarzschild spacetime, for a black hole that does not spin. The spin of the black hole, is what causes this so called frame dragging. As an example, the orbit of the earth around the sun, is a near circular ellipse. The constellations of the zodiac, are the stars on the far side of the earths orbit during that month. The path of the earths orbit traces a two dimensional plane, known as the ecliptic plane. For our rotating black hole scenario, this is no longer the case, as any non equatorial orbit about the black hole, tilts. That is to say that the ecliptic plane of our orbit is shifting like that of a spinning top on the table. This is even more wild than anyone could ever imagine!

Now imagine we have a handful of rotating black holes, all orbiting through a common spacetime, with their own masses and spins. Black holes spiral inwards towards one another and merge, releasing gravitational waves. These tremors in spacetime, then bend around other nearby black holes, which are merging themselves. Its just a gigantic mess of curved geometries. After a while, everything settles down to a few large black holes, with higher masses and new spins, all moving away from one another. In the theory of general relativity, matter tells spacetime how to curve, yet spacetime tells matter how to move. The matter we are referring to are the black holes themselves. Note that it is not just space or time itself that is bending, but our full spacetime geometry. This is the worldview we get, after unifying special relativity with universal gravitation.

Here is the usual definition for the radius. Note that to avoid confusion with the radius for a spinning geometry, we redefine the usual non-spinning radius as capital R, leaving the lowercase r to be defined next.

$$R=\sqrt{x^2+y^2+z^2}$$

```
R = sqrt{ x^2 + y^2 + z^2 }
```

This equation is a new definition for the radius, which also depends on the spin parameter "a". The lookback time then becomes the usual definition of the radius function divided by the speed of light. This is only natural in oblate spherical coordinates, and will be used in the following formulas.

$$r=\sqrt{\frac{R^2-a^2+\sqrt{(R^2-a^2)^2+4\,a^2\,z^2}}{2}}$$

```
r = sqrt{frac{R^2-a^2+sqrt{(R^2-a^2)^2+4 a^2 z^2}}{2}}
```

Remember the gravitational potential from earlier. Here is the same thing, but with spin included. Sometimes called the Appell potential for the electric potential field around a loop of wire, the same applies to gravity. Since in cartesian coordinates, the point particle singularity is an extended ring, sometimes called a ringularity. The gravitational potential around such an extended ring is what this equation defines.

$$H=\frac{2GM\,r^3}{c^2\left(r^4+a^2\,z^2\right)}$$

```
{ H = frac{2GM r^3}{c^2left({r^4 + a^2 z^2}right)} }
```

Like before, alongside the gravitational scalar potential, is the gravitational vector potential. It tells us the paths of light-rays that are directed straight down towards the singularity. However as we know, the spin of the hole, creates a frame dragging effect, that spins the paths off the past light cone. Nonetheless, the vector potential still traverses the lookback time down to the ringularity. If we had a camera at the ringularity, every point on the celestial sphere, would be an oblate spherical projection, and the vector potential will just be a incoming light ray at that specific angle.

$$l_\mu dx^\mu=c\,dt+\frac{r\left(x\,dx+y\,dy\right)}{r^2+a^2}+\frac{a\left(y\,dx-x\,dy\right)}{r^2+a^2}+\frac{z}{r}dz$$

```
{l_%mu dx^%mu = c dt + {frac{r left({x dx + y dy}right) }{r^2 + a^2}} + {frac{a
left({y dx - x dy}right)}{r^2 + a^2}} + {frac{z}{r}} dz}
```

From here, one can define the kerr metric geodesics, the equations we have used in the section on the schwarzschild metric geodesics remain unchanged, the only difference is that instead of using the schwarzschild metric, one uses the kerr metric. So the equations above suffice to give us the geodesic equations for spinning and non-spinning particles that obey both special relativity, and universal gravitation. Now that we have the orbits around a rotating black hole, its time to investigate what happens when one includes spin to the klein gordon equation.

Section 4.2: Proca Equation

The proca equation, has the same plane waves as the klein gordon waves, which means it decribes a relativistic quantum particle. But unlike the klein gordon equation that acts on a scalar field, the proca equation acts one a 4-vector field. We all know about scalar fields, as they're invariant under lorentz transformations. Remember how the position 4-vector transforms in special relativity. Yes, exactly! That is how the 4-vectors of the proca equation transform. Nothing new again, just the objects that we know from before.

Why a 4-vector, and now a 3-vector? The answer is simple, in that one is dealing with special relativity. This means that Lorentz boosts will skew a three vector which is incorrect, but an actual 4-vector, like the t,x,y,z components of 4-position transform correctly under all lorentz transforms. Why? If one took the space-time interval, of all 4-vectors, one would always get an invariant number. The klein gordon equation is a field of such numbers, and is invariant. So for the property to extend, one must also make sure that whatever one uses to build a field theory, that its quantities reduce to the klein gordon scalar when one applies the space-time interval to it. And a 4-vector does exactly that!

What makes a 4-vector field interesting is not just that it transforms under lorentz transformations, but that in the limit where the mass vanishes, one recovers Maxwell's equations without sources or currents. This means that in the massless limit, this proca equation can be used to explain the wave-function for photons of light. More on that later on. For now lets get straight to the math.

Here is the proca equation written out:

$$\eta^{\nu\sigma}\partial_\nu\partial_\sigma\Psi_\mu-\frac{M^2c^2}{\hbar^2}\Psi_\mu=0$$

```
{ %eta^{%nu%sigma} partial_%nu partial_%sigma %PSI_%mu   } - frac{M^2 c^2}
{hbar^2} %PSI_%mu = 0
```

Here we see its just four klein gordon equations side by side. Good, we have a 4-vector we can build at every spacetime event. But now we have a problem too… The relative values of the four components can take on any value. That is a problem because we want the values to be constrained by special relativity. This makes mure sense when one decomposes the full 4-vector field into plane waves.

Like the klein-gordon case, each plane wave is a lorentz transformed version of any other of its plane waves. For the proca case, doing a lorentz transform should also transform the components of the 4-vector accordingly, and its freely chosen. Therefore we have to impliment a constraint that aligns the polarization 4-vector values to point to a constant phase in each plane wave, which means that the polarization and linear momentum 4-vectors must be perpendicular. This is the constraint that makes this work.

$$\eta^{\mu\nu}\partial_{\mu}\Psi_{\nu}=0$$

```
{ %eta^{%mu%nu} partial_%mu { %PSI_%nu  } } = 0
```

One might still ask, why spin 1? For the quantum wave-function, spin is not a rotation, but a direction. For a classical particle, yes its intrinsic angular momentum would be how fast its spinning, like a spinning top. However for its quantum counterpart, the intrinsic angular momentum, or spin, would take on integer and half integer values in units of the reduced planck constant hbar. So for an extremely high spin, it would reduce to the statistics of a spinning point particle. But for lower spins like spin 1, the 4-vector works, and for spin 0, a simple number would work. For even higher spins like spin 2, we would have the tetrad, which is a Jacobian matrix that converts between two 4-vector bases. Such a matrix could be built from a spin-2 relativistic wave equation that takes the Kronecker product between two normalized 4-vectors. For spin 3, one would have a 4 by 4 by 4 tensor, or data cube of a wavefunction, that is built from the Kronecker product between three 4-vectors. And so on for spin 4 and spin 5. In other words, the greater the spin, the more the wave-function changes under a change in perspective, say a Lorentz transformation. So for spin 1, its a 4-vector field. However, the idea now, is to couple the proca equation to its own rotating black hole solution: The kerr metric for spin one, in the cartesian Kerr-Schild coordinate system!

Section 4.3: Proca on its Kerr Metric

In the spin one case, both the Kerr black hole and the Proca force waves have to have the same spin. For the proca equation, it is naturally spin one. However the geometry of the Kerr spacetime, which describes black holes with intrinsic spin, the spin can take on any value. Here, we just fix it to be spin one, and then fix it into dimensions of distance.

$$a=\frac{\hbar}{Mc}$$

```
a = { frac{hbar}{Mc} }
```

Here is the equation for the proca equation in its kerr metric.

$$g^{\nu\sigma}\partial_{\nu}\partial_{\sigma}\Psi_{\mu}+\left(\partial_{\nu}g^{\nu\sigma}\right)\partial_{\sigma}\Psi_{\mu}-\frac{M^2c^2}{\hbar^2}\Psi_{\mu}=0$$

```
{ g^{%nu%sigma} partial_%nu partial_%sigma %PSI_%mu   } + {  left(  partial_%nu
{g^{%nu%sigma} }right) partial_%sigma %PSI_%mu    } - frac{M^2 c^2}{hbar^2}
%PSI_%mu = 0
```

Along with the constraint.

$$g^{\mu\nu}\partial_{\mu}\Psi_{\nu}+\left(\partial_{\mu}g^{\mu\nu}\right)\Psi_{\nu}=0$$

```
{ g^{%mu%nu} partial_%mu { %PSI_%nu  } +   left({partial_%mu g^{%mu%nu} }right)
%PSI_%nu } = 0
```

As we can see here, the cartesian Kerr-Schild coordinates, simplify matters drastically. There is no torsion, so the covariant derivatives inside the field strength tensor become only partial derivatives. When expanding out the constraint equation from the field strength tensor, space-time is vacuum, so the ricci tensor vanishes. Then, because the determinant of the cartesian Kerr-Schild coordinates always has a constant 4-volume, means that one can write it without the christoffel symbols and use its divergence instead. This gives the form shown above.

Section 4.4: Kerr Tetrad Geodesics

I looked online, and read up on countless articles on black holes. Everything about black holes, was mostly dusty old papers on numerical relativity. Then, something caught my eye: The Lorentz Covariant Treatment of the Kerr-Shild Geometry. A paper in 1975 by Gurses and Gursey. It said there exists a coordinate system where the geometry separates from the gravity. Without further research, I set out to test whether this so called Kerr-Schild geometry was the key. I ran a MATLAB script that shows that the kerr-schild coordinates do indeed work from a battery of tests that I ran. Therefore, by knowing that kerr-schild coordinates do indeed separate the gravitational field from the background spacetime, means that it also proved that black holes are particles.

A tetrad is a set of basis vectors that transforms you from one set of coordinates to another. In this case, it creates the gravitational field out of the relativistic vacuum. But the battery of tests I ran in MATLAB showed that it is the only tetrad that is kerr-schild, lorentz covariant, represents a particle, and obeys general relativity.

This equation is defines the tetrad, which is a field of inertial spacetime frames. It is the most important equation for the whole book. The reason isn't just that tetrads allow for coupling to fermionic fields, but that this tetrad is the only tetrad that is both globally Lorentz covariant, and resides in the cartesian Kerr-Schild coordinate system. This form is what is required to turn a singularity for a rotating black hole from a pathology that cannot be renormalized away, into a simple trajectory of a particle.

$$e^{A}_{\mu}=\delta^{A}_{\mu}+\frac{H}{2}l^{A}l_{\mu}$$

```
e^A_%mu = %delta^A_%mu + {frac{H}{2} l^A l_%mu }
```

Since the lorentz transformations and with its kerr-schild add-on, are defined as tetrads, one can find the curved spacetime interval g from the flat spacetime interval eta by defining the tetrad to give the orthogonal basis vectors for the curved geometry within the background minkowski spacetime geometry. This is important for calculating the metric tensor from the tetrad, and tells us that my tetrad is a kind of creation operator that creates a rotating black hole from the flat space-time vacuum when applied.

$$g_{\mu\nu}=\eta_{AB}e^{A}_{\mu}e^{B}_{\nu}$$
```
g_{%mu%nu} = %eta_{AB} e_%mu^A e_%nu^B
```

Since we have a tetrad that takes us from flat to curved, one might wonder if there's an inverse. Yes there is, and it takes you back from curved to flat. You can think of it as "undo-ing" the creation of a black hole, flattening it back out to Minkowski space-time.

$$e^{\mu}_{A}=\delta^{\mu}_{A}-\frac{H}{2}l_{A}l^{\mu}$$
```
e_A^%mu = %delta_A^%mu - {frac{H}{2} l_A l^%mu }
```

Since the tetrad converts between flat and curved inertial frames, we need an inverse tetrad. This can be achieved by multiplying it by the metric tensor we defined earlier in the book, but using the new definitions for the radius, potential, and vectors. We can also undo this operation, now by multiplying again not by the usual kerr metric, but with the inverse kerr metric. This inverse metric can also be defined using the inverse tetrad as shown here.

$$g^{\mu\nu}=\eta^{AB}e^{\mu}_{A}e^{\nu}_{B}$$
```
g^{%mu%nu} = %eta^{AB} e^%mu_A e^%nu_B
```

Now we move on to the spin half case. Like before with the proca equation, the dirac equation, extends the klein gordon equation to include spin. And such an equation when coupled to its own grvavitational field requires tetrads, not just the metric tensor. Lets now move on to this equation…

Section 4.5: Dirac Equation

In 1928, Paul Dirac found another relativistic wave equation, one that extends the Klein Gordon equation, to include a new phenomenon, spin! The spin of the Dirac waves, is due to a new direction that these waves can vibrate in. In some sense, the more ways in which these quantum waves can vibrate, the greater the spin. Spin can only increase or decrease by half steps. In this case, we have a spin up and spin down electron, along with a spin up and spin down positron. Each and every component, are themselves Klein Gordon waves. It is only when we change our point of view, that the components mix into one another. For example, take a Klein Gordon plane wave vibrating purely as a spin up electron. If we rotate this wave, and look at it from another angle, the plane wave, with everything else being the same, is now vibrating as a spin down electron. Similarly, if we boost our whole setup so far, the vibration in the first two matter electron components, is now slightly vibrating the last two antimatter electron components also, also known as the spin up and down positrons. We say that Klein Gordon waves and Dirac waves, have the spin that they have based on this simple idea. Klein Gordon waves have a spin of zero, because if we rotate the setup by any angle, even after many turns, everything stays the same. This is obvious. However, if we rotate a Dirac wave by a full turn, the components mix only half of what we expect. This means that it takes two full turns of a

Dirac wave to return to its initial configuration. We say that a Dirac wave has a spin of a half, for this very reason.

Now imagine we have a bunch of Klein Gordon and Dirac waves, all undulating independently throughout spacetime. Every quantum wave, vibrates in its own abstract dimension, with every dimension corresponding to the probability coordinates of every marble. This is similar to how the spin of all these marbles acts like. The Klein Gordon waves, overlap and build up, whereas the Dirac waves interfere and sometimes cancel out. In this theory, it is all about a new entity called a relativistic quantum field. Imagine a crystal array of quantum marbles that extends out to infinity in all directions. If we hit this crystal with a hammer, it will vibrate just like a big block of rubber. As it turns out, these vibrations are completely identical to our Klein Gordon and Dirac waves. Since these vibrations are nothing but sound waves living in a crystal lattice made of quantum balls and quantum springs, there should be a minimum unit, or quantum of a field. As luck would have it, every field quantum acts independently, just like a marble. Therefore our system of a bunch of Klein Gordon and Dirac waves, is thus nothing more than a collection of field quanta. This is our worldview after unifying special relativity with quantum mechanics.

This is the 4 by 4 matrix definition for the t direction. As we can see it only has real valued components. This matrix points into the direct future.

$$\hat{t} = \begin{matrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{matrix}$$

```
hat t = { matrix{0 # 0 # 0 # -1 ## 0 # 0 # 1 # 0 ## 0 # -1 # 0 # 0 ## 1 # 0 # 0
# 0} }
```

This is the 4 by 4 matrix definition for the x direction. Being real valued, it points to the right.

$$\hat{x} = \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{matrix}$$

```
hat x = { matrix{1 # 0 # 0 # 0 ## 0 # -1 # 0 # 0 ## 0 # 0 # 1 # 0 ## 0 # 0 # 0
# -1} }
```

This is the 4 by 4 matrix definition for the y direction. Being real valued, it points to the front.

$$\hat{y} = \begin{matrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{matrix}$$

```
hat y = { matrix{0 # 0 # 0 # 1 ## 0 # 0 # -1 # 0 ## 0 # -1 # 0 # 0 ## 1 # 0 # 0
# 0} }
```

This is the 4 by 4 matrix definition for the z direction. Being real valued, it points to the top.

$$\hat{z}=\begin{matrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{matrix}$$

```
hat z = { matrix{0 # -1 # 0 # 0 ## -1 # 0 # 0 # 0 ## 0 # 0 # 0 # -1 ## 0 # 0 # -1 # 0} }
```

Now, because we have 4 by 4 matrices of spacetime itself, using the imaginary unit by itself will add unecessary complexity of using complex numbers, which would introduce electric charges, which don't exist here, because charges don't input into the Einstein-Cartan theory. Instead we redefine the imaginary unit as another gamma matrix, sometimes called the 4-volume element or fifth gamma, as it handles integration in spacetime algebra.

$$i=\hat{t}\,\hat{x}\,\hat{y}\,\hat{z}$$

```
{ i = hat t hat x hat y hat z }
```

This is where the magic happens. This is the gamma vector. The index "mu", is now an actual spacetime 4-vector. But each term is not just a number, but a 4 by 4 matrix in polarization space. Think of it like the definition of what t,x,y,z, all are without knowing what the Minkowski metric is. By multiplying together different gamma matrices, we get new gamma matrices that depend on which two matrices you multiplied and in which order. We can say that the gamma matrices are the square root of the Minkowski metric.

$$\gamma^A=[\hat{t},\hat{x},\hat{y},\hat{z}]$$

```
{ %gamma^A = [hat t,hat x,hat y,hat z] }
```

Now we get to the quantum matter waves. This equation is the definition for the Dirac operator. Like the Klein Gordon operator, the Dirac operator is a kind of square root of the Klein Gordon operator. The alpha and beta indeces are not space-time vector indeces but a more abstract spin space, sometimes called polarization space. If one took this operator and squared it, the cross terms would cancel out, leaving the gamma matrices that will anticommute to give the Minkowski metric, recovering the Klein Gordon operator.

Here is the full Dirac equation. The wavefunction PSI, has its usual definition, as overlapping two wavefunctions, one initial, and one final, to give a total probability of such a transition to occur. As one might see with the polarization label beta, that it's actually 4 seperate equations side by side. Its the way the cross-terms interact between the equations that is the reason for its structure obeying special relativity.

$$\gamma^A\partial_A\Psi+i\Psi\frac{Mc}{\hbar}=0$$

```
{ %gamma^A }partial_A %PSI  + i %PSI{frac{M c}{hbar} } = 0
```

Now with our dirac equation in our posession, its finally time to derive the full CGH equation with spin half. We did spin one, and saw how intricate it is. Given that spin half is even more tedious, if you bear with me, you will then have an equation that can predict both higher spins and their limiting cases of CGH physics. This is why I saved this beast for last. Lets now tackle it head on!

Section 4.6: Dirac on its Kerr Tetrad

In the spin half case, both the Kerr black hole and the Dirac matter waves have to have the same spin. For the dirac equation, it is naturally spin half. However the geometry of the Kerr spacetime, which describes black holes with intrinsic spin, the spin can take on any value. Here, we just fix it to be spin half, and then fix it into dimensions of distance.

$$a=\frac{\hbar}{2Mc}$$

```
a = { frac{hbar}{2Mc} }
```

Here are the massive dirac equations using the real majorana gamma matrices for a space-time algebra in the curved spacetime of the kerr geometry in cartesian kerr-schild coordinates with rotating black hole mass M and spin J equal to the particle mass m and spin s respectively, with M being a free parameter alongside the coordinates t,x,y,z, for a wavefunction of a given fixed spin of s*hbar, where s=1/2:

$$e^{\mu}_{A}\gamma^{A}\partial_{\mu}\Psi+\frac{1}{4}e^{\mu}_{A}\gamma^{A}\Psi\left[\partial_{B}e_{\mu A}-\partial_{A}e_{\mu B}\right]\left(\gamma^{A}\gamma^{B}-\gamma^{B}\gamma^{A}\right)+i\Psi\frac{Mc}{\hbar}=0$$

```
 { e^%mu_A %gamma^A }partial_%mu %PSI + {frac{1}{4}} {e^%mu_A %gamma^A } %PSI
left[{partial_B e_{%mu A} - partial_A e_{%mu B}}right]  left({%gamma^A %gamma^B
- %gamma^B %gamma^A}right)  + i %PSI{frac{M c}{hbar} } = 0
```

Here it is! The full CGH physics equation for spin half. One can use it to rederive any of our previous equations through the Bargmann-Wigner formalism. Two of these equations in parallel with twice the spin parameter gives us the proca or spin one version. In fact, we will use precisely this equation to derive the theory of everything. So lets go ahead and do exactly that.

Chapter 5: Building a Theory of Everything

To even attempt a unification, I came up with my quantum relativity framework. It states that reality is made of classical, relativistic, fields. Classical in that you do not apply quantum mechanics to anything, relativistic in that the klein gordon equations are used, which have Lorentz invariance. A field is just a mathematical object that takes a value at every event in space-time. Put together, the klein gordon and other Relativistic Wave equations are what reality is, and it says nothing about particles.

I have seen a similar treatment by Rodney Brooks, in his masterpiece known as "Fields of Color", he lays out the standard model for a lay audience, and the notion that the quantum field theory community has gone astray, restoring it would require a new framework. Something I already had in place: My quantum relativity framework. This is why I will explain why fields are fundamental and what the theory of everything is.

Section 5.1: Poincare Symmetry of Minkowski Spacetime

The whole universe, one way or another, obeys Poincare symmetry. Poincare Symmetry are the 10 transformations of space-time, that leave the Minkowski space-time interval invariant. We glossed over it in section 3.4, where we unified special relativity and universal gravitation for a single particle without spin. In this section, we will explain why it is so important and how it works.

The importance of Poincare symmetry is due to its wide variety of applications. Special relativity which describes how matter behaves in such a spacetime, has radically changed our newtonian worldview where time and space were absolute, now they depend on how you are moving. This means that any theory we build, has to begin with space and time, see what its symmetries are, and then move on to describe matter in it later.

This is because when we incorporate special relativity into both quantum mechanics and universal gravitation, which as your remember is the hydrogen atom orbitals for both bound and scattering states, we had to abandon the simple absolute space and time, and this mean't abandoning the physics that works with those Galilean symmetries. That is why the previous chapter got so difficult, and is due to the fact that Poincare symmetry reduces to Galilean symmetry for speeds much slower than the speed of light.

However, once we included Poincare symmetry first, we automatically saw that the klein gordon equation and the schwarzschild metric geodesics are also incomplete, because the symmetry of Minkowski space-time allows spin to exist. That is why we developed the spin 1 and spin $\frac{1}{2}$ equations, because leaving them out, would be limiting to a universe that might have such symmetries.

In fact today we know that fundamental particles follow a wigner classification of mass and spin casimirs, precisely the properties of any matter in special relativity. That is why we will begin with the symmetry underlying the space-time of special relativity, because it is absolutely rock bottom essential to both the physics and the spacetime the physics lives in. And when we get to the black hole particle duality, we will use these symmetries to justify why the choices we made for equating the mass and spin parameters for both black holes and relativistic wave equations is correct.

Remember way back in the beginning of the book, when we talked about 2D polygons and how a rotation is just a matrix multiplication with sines and cosines. Well the same is true for a lorentz boost, except one just replaces the usual trig functions with their hyperbolic counterparts. And given that all trig functions including the hyperbolic trig functions are all built from a matrix exponential, with this in mind, one can built an arbitrary lorentz transformation with the following matrix exponential.

This equation is the definition for the Lorentz transformations. As you can see, boosts are symmetric, but rotations are anti-symmetric. This ultimately stems from the fact that the time dimension rotates in an opposite way than the three spacial dimensions rotate. This in turn happens because all of the Lorentz transformations keep the full Minkowski spacetime interval the same between spacetime frames.

$$\Lambda = expm\begin{pmatrix} 0 & K_x & K_y & K_z \\ K_x & 0 & -J_z & J_y \\ K_y & J_z & 0 & -J_x \\ K_z & -J_y & J_x & 0 \end{pmatrix}$$

```
%LAMBDA = expm left({ matrix{0 # K_x # K_y # K_z ## K_x # 0 # -J_z # J_y ## K_y
# J_z # 0 # -J_x ## K_z # -J_y # J_x # 0} }right)
```

In fact, notice that K is a rapidity 3-vector for Lorentz boosts, where as J is a angular 3-vector for spacial rotations. This is because of minus signs between opposing J's and the lack thereof for opposing K's. The reason is simple. We are keeping the Minkowski interval invariant, so if one takes a space-time interval of the transformed frame, one must get the same answer in the untransformed frame. Here is the defining relation.

Here is the reason for the invariance. And it's not just light-speed that travels at the exact same speed, but the full spacetime interval. We can clearly see that the Lorentz transforms themselves form a kind of inertial spacetime frame. Having all spacetime frames related by Lorentz transforms, means that spacetime intervals in each frame will produce the same answer across frames. This will be important for later when we derive the tetrad which is a type of spacetime frame used for a curved spacetime geometry.

$$\Lambda^{\nu}_{B}\eta_{\mu\nu}\Lambda^{\mu}_{A} = \eta_{AB}$$

```
{%LAMBDA_B^%nu %eta_{%mu%nu} %LAMBDA^%mu_A =  %eta_{AB} }
```

And for completeness, here is the space-time interval we defined earlier in the book. It is also known as the Minkowski metric tensor. The reason for the hyperbolic

trig functions, the imaginary nature of time, the constant speed of light, etc… Is all because of this simple minus sign for the time-time component of the Minkowski metric shown here. If one applied spacetime 4-vectors to it, we would arrive at the flat spacetime interval. But here, we have the bare bones 4 by 4 matrix version. Clearly, time rotates differently, because it squares to a negative distance. Time is an imaginary dimension.

$$\eta_{\mu\nu} = \begin{matrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

```
%eta_{%mu%nu} = { matrix{-c^2 # 0 # 0 # 0 ## 0 # 1 # 0 # 0 ## 0 # 0 # 1 # 0 ##
0 # 0 # 0 # 1} }
```

In fact now that we have defined the space-time background in which the universe lives in, it is time to start re-building the quantum and gravitating forms of matter within. And we will soon see how similar both descriptions are. Start with a spacetime, find the symmetry invariants, build the resulting quantum gravitational hydrogen-atom equation for such a "particle" with those symmetries intact, and then extend to a mutliple particle system, and lastly solve the equations. In other words, begin with absolutely nothing: Minkowski spacetime, add matter or content in the form of the Black Hole – particle duality, and solve the equations. One of these solutions is our universe and known as the standard model of particle physics. With its lagrangian as the input to our duality, we have built our black hole particle duality.

Now, the symmetries of Minkowski spacetime, when applied to Universal Gravitation gives us a way to build the field equations for a force field such as gravity. The trick is to make a set of symmetries local instead of global. In other words we are looking for a theory in which space-time is no longer Minkowski everywhere and for all time, but locally, when zoomed in to a small patch of this larger curved geometry, Minkowski is recovered. Such spacetime curvature by defintion is gravity. Since we are interested in Poincare symmetry, and not other symmetries, we will restrict ourselves to the vacuum case. So here I present to you the next section:

Section 5.2: Vacuum Einstein Equations

Our two worldviews, one of the heavy, and the other of the light, both share some common features. Black holes and quantum waves, both have mass, and also both have spin. Because of this fact, a setup of the exact same physical system in one worldview, will mimic the behavior in the other worldview. It turns out that the atoms and subatomic particles, of which we are made, are not marbles at all. They are instead a kind of quantum black hole. A quantum black hole, behaves more like an actual black hole if it is really heavy, yet also acts like a quantum wave if it is really light. Therefore, in order to understand this, we needed to unify the three pillars of physics into a single framework. This means that everything is one. Literally!

At this point, we have all of the pillars of physics we need, in order to unify all of physics into the theory of everything. However, such a unification of physics will require two radical paradigm shifts. The first idea is all about special relativity, and how to unify it with the rest of physics. The second idea is to reconsider the properties of matter, such as mass and spin. Lets review some key ideas in special relativity before we move on. The spacetime of special relativity, also known as Minkowski spacetime, replaces the idea of absolute time, with the idea of an absolute light speed. With this idea in mind, the symmetries of space and time, unify into a structure known as spacetime.

The spacetime of special relativity, called Minkowski spacetime, has certain symmetries, known as Lorentz Transformations. They keep the speed of light constant, under a change in perspective: The relativity principle. They are the symmetries of spacetime itself: 3 Hyperbolic Boosts, 3 Spacial Rotations, 4 Spacetime Translations. From these symmetries, emerges new phenomena.

In quantum field theories that involve relativistic wave equations, which live on Minkowski spacetime, the mass and spin of these field equations, are a direct result of spacetime symmetries. Mass is the minimum frequency for a wavepacket at rest. Spin is the amount of change between field polarizations under a change in spacetime perspective. Due to the quantum principle, the mass and spin of a wavepacket, when zoomed far enough away, appears to act like a spinning top, with the mass being the top's inertia, or resistance to changes in motion, and its spin being its angular momentum while spinning.

However, all relativistic wave equations, still live on flat Minkowski spacetime. Sure, there could be interactions between fields that give rise to non-gravitational forces. This is what the standard model does, as it is a quantum field theory, that maps out the possible interactions between wavepackets. However, in Einstein's General Theory of Relativity, gravity is the result of the curvature of spacetime itself. Therefore in General Relativity, spacetime is a classical field theory in its own right. The reasoning to this approach is astonishing, because one has to invent a new way to produce classical field theories involving symmetries.

The answer is a new kind of symmetry known as local symmetry. So far, relativistic wave equations had global spacetime symmetries: Mass and Spin. However local symmetries are symmetric only in small regions of the field spaces. We already encountered local symmetries in the standard model. They were not local spacetime symmetries, but local field symmetries. However, one might ask what happens when one makes spacetime symmetries local in their own right. The answer: General Relativity!

Okay not quite, as I must admit. Although General Relativity is a classical field theory, where spacetime curvature gives rise to gravity. It doesn't have all of the local symmetries of spacetime. Only spacetime translations become local symmetries in Einstein's theory. However, what we are asking is what happens when all spacetime symmetries become local symmetries. In the general case scenario, one must intoduce a

new concept known as torsion… Just like energy density currents and momentum density currents couple to gravity, spin densities couple to torsion. And, just like how the curvature of spacetime is the definition of gravity, torsion has to be included to make sure the boost and rotation symmetries are also local in nature. The result is a new theory called: Poincare gauge theory, or gauge theory gravity, which has field equations known as the Einstein Cartan Theory.

However, in our theory, we are not to couple the fields of the standard model to the einstein cartan theory. And here's why? Solutions of the einstein cartan theory produce black holes, whether or not we couple other fields to the theory. I mean why not let everything be fields, including spacetime curvature and torsion, and what is the deal with black holes? This is a bug, not a feature. Black holes have to be associated with point particles… And as we know, point particles do not exist. At least in classical field theory. But, in this oberservation comes a truly surprising result.

As we know we applied quantum physics to a single point particle, and recovered a classical field theory of relativistic wave equations. The massive Klein Gordon and massive Dirac equations respectively. Therefore, if black holes are also points, means one thing: Every rotating black hole with a given mass and spin, must be associated with a relativistic wave equation that describes a single quantum particle! And, in our einstein cartan theory, which we just coupled it to the standard model field theory, and that doesn't do it justice. Black holes exist regardless of field content. What we need is a theory where black holes exist, only when the fields are vibrating, and when no fields are vibrating, there shoudn't be any black holes present. Moreover their mass and spin should also be completely identical. How to do this?

Since Minkowski spacetime has the same global symmetries, even in the case with the Einstein Cartan theory, there shoudn't be any extra local symmetries. By coupling the standard model to it, we included plenty more extra symmetries that the Einstein Cartan theory couldn't handle. Let me say it backwards: All local symmetries have to have global symmetry counterparts. And in the standard model, we only have a global spacetime symmetry, there are no global field symmetries. Therefore, lets keep the standard model and the Einstein Cartan theory separate, for now.

Thus, in the vacuum, the Einstein Cartan theory, reduces back to good old General Relativity. How? Without any mass or spin present from other field theories, the right hand side of the Einstein Cartan theory vanishes. Spin vanishes, and so does the mass. Torsion is algebraic, so it completely vanishes without spin, leaving us with general relativity. But in general relativity, the energy momentum tensor is also all zero, due to vanishing mass. Therefore, one is left with general relativity without any sources.

In vacuum general relativity, there are plenty of solutions. One can consult a numerical relativity simulation, just to be sure… rotating black holes orbit and collide, gravitational waves propagate around everywhere. And, everything is just vacuum. So

clearly, one has sources without a source, particles from vacuum, or otherwise put: Everything from nothing!

These black holes have only mass and spin, why not include electric charge too? Electric charge assumes the Einstein-Maxwell system, which means photons of light must couple to charged black holes. However photons have a U(1) gauge symmetry that couples to the complex phase of the wavefunction in question. Spacetime does not contain U(1) as a symmetry, only the O(3,1) Lorentz boosts, spacial rotations, and space-time translations of the Poincare group. Therefore if one wants a purely Poincare Gauge theory of the vacuum, one must exclude all charges except from mass and spin, the poincare casimirs of the spacetime vacuum. This doesn't mean that charge doesn't exist, it just won't couple to the black hole solutions and as such the gravitational field would only "know about" mass and spin, giving rise to gravity and torsion respectively. The Kerr solution is thus the most general vacuum solution of the Poincare gauge theory for gravity.

What we need now, is a way to directly couple these rotating black hole solutions, to our relativistic quantum particles. We need a way to produce the spacetime geometry straight from its solutions, and not the other way around.

Since fields are the ontology, lets go through our first example of particles emerging from fields. In standard textbook general relativity, spacetime and the gravitational field are the same curved continuum, and black holes are treated as pathological monsters. But in this book, we already know a choice of coordinates that does away with all of this: It makes general relativity exactly linear. Spacetime separates from the gravitational field, and as such, black holes are no longer pathological… They are particles!

What gets missed in this discussion of what I call the black hole particle duality is that even though we singled out a set of coordinates, the black hole equations are still solutions of general relativity. Therefore, we didn't kill general relativity. What we did was discover a coordinate system within general relativity with some interesting properties. Since we are still obeying general relativity, which is a classical field theory of the gravitational field, means one thing… The particle solutions describing black holes emerged from a field theory of gravity. We have our first example of particles emerging from a field! So General Relativity gives black holes as particle solutions directly. Now since general relativity is a field theory, it must have field equations it obeys. Here they are:

The first step to the full equations that space-time obeys, is to define what are known as the Christoffel symbols. They are commonly used in finding geodesics or paths of inertial motion through a curved spacetime geometry. But their general form allows one to compute parallel transport of any 4-vector through a given spacetime. Here they are:

$$\Gamma^{\lambda}_{\mu\nu}=\frac{1}{2}g^{\lambda\rho}\left(\partial_{\nu}g_{\rho\mu}+\partial_{\mu}g_{\rho\nu}-\partial_{\rho}g_{\mu\nu}\right)$$

```
{ %GAMMA^{%lambda}_{%mu%nu} = {frac{1}{2}}{g^{%lambda%rho}} left({partial_%nu
g_{%rho%mu} + partial_%mu g_{%rho%nu} - partial_%rho g_{%mu%nu} }right) }
```

Next, is the calculation of the Riemann curvature tensor, and such a curvature can be computed by parallel transporting a vector through a parallelogram. Since curved space-time geometries always change the parallel transported vector from its original values around, one needs a very heavy object to handle it. Here it is:

$$R^{\rho}_{\sigma\mu\nu}=\partial_{\mu}\Gamma^{\rho}_{\nu\sigma}-\partial_{\nu}\Gamma^{\rho}_{\mu\sigma}+\Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma}-\Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}$$

```
{ R^%rho_{%sigma%mu%nu} = partial_%mu %GAMMA^%rho_{%nu%sigma} - partial_%nu
%GAMMA^%rho_{%mu%sigma} + %GAMMA^%rho_{%mu%lambda} %GAMMA^%lambda_{%nu%sigma} -
%GAMMA^%rho_{%nu%lambda} %GAMMA^%lambda_{%mu%sigma}}
```

Then we can sum over a spacetime index of the curvature tensor to obtain the Ricci tensor. We do this by summing over mu. This is our first tensor to be used in our field equations.

$$R_{\nu\rho}=R^{\mu}_{\nu\mu\rho}$$

```
{ R_{%nu%rho} = R^%mu_{%nu%mu%rho} }
```

And after that, we can contract with the metric tensor we began with. This gives us the Ricci scalar. It is the only term of the Einstein-Hilbert action. The resulting extremum of this action is given by the Einstein Field Equations for the vacuum.

$$R=g^{\mu\nu}R_{\mu\nu}$$

```
{ R = g^{%mu%nu} R_{%mu%nu} }
```

With these components, one now has what is needed for the vacuum Einstein field equations, defined below:

$$R_{\mu\nu}-\frac{1}{2}R\,g_{\mu\nu}=0$$

```
{ R_{%mu%nu} - {frac{1}{2}} R g_{%mu%nu} }=0
```

What should become clear is this: Although this field theory describes gravity and spacetime, the particle solutions, when quantized, don't give the same equations. If they did we would have a graviton particle with spin 2 and massless. Gravitons would as such destroy our coordinate system and with it our theory. But by quantizing any other field theory, the resulting particles do actually give the same equation.

It is this fact that allows one to only apply quantum physics to our black hole particle solutions of the gravitational field and not to spacetime itself. If one did try and quantize the full nonlinear beast of general relativity, we would end up with string theory or loop quantum gravity, which is not only beyond our scope of topics, but is also wrong, because they contain gravitons which can never be detected. By quantizing our black hole particles, and not general relativity, we circumvent a critical issue in modern physics: Quantum Gravity! So lets now move on from this quantum gravity nonsense and quantize an actual field theory that does have a particle interpretation that actually works… So General Relativity gives black holes as particle solutions directly. Now since general relativity is a field theory, it must have field equations that its black holes obey.

Usually, they are defined with matter present. But we don't need to couple it to anything. Without a spin density, torison as we noted earlier vanishes identically, giving us the left hand side of the Einstein equations for general relativity exact as is. Then, without a mass density, the right hand side, which is usually given as the stress-energy tensor, goes to zero. Why this form?

Because, since we are treating black holes as particles, and since black holes occur naturally in a given spacetime, means we do not need to introduce extra matter variables. Stated more concretely, the Einstein Field equations for a vacuum spacetime, when solved exactly, give the spacetime geometry for a set of black holes. They have the same space-time symmetries of Poincare, that in the duality, give rise to particles with mass and spin. Since the most general black holes are uniquely described by only mass and spin, these are the equations we are to solve, as introducing matter will only increase the number of parameters that define such entities, like electric charge for example. Since space-time is described by orthogonal 4-vectors that are real valued, and not complex valued, means that space-time must be electrically neutral, and given this, the black hole solutions must also be neutral. This can only be true if space-time is vacuum. Moreover, it is quite amazing that Einstein uncovered this in 1915, only ten years after his Special Relativity.

Section 5.3: A Black Hole – Particle Duality

Did you know that all particles such as electrons of electricity, photons of light, even quarks, are all rotating black holes. A black hole is a region of space in which gravity is so strong that light cannot escape it's deadly grip. A black hole that spins like a top on the table is known as a rotating black hole. The only difference between a black hole and a particle is size. Heavy particles act more like black holes, and light black holes act more like particles, because they are the same mathematical object.

All fundamental particles are black holes and all black holes are fundamental particles. They both share the exact same mass and spin. They also both live in the cartesian Kerr-Schild coordinate system that allows the separation of the gravitational field from the Minkowski background spacetime. The geometry is given by the Kerr solution of vacuum General Relativity, which describes a rotating black hole. Electric and other charges are set to zero, as they do not exist as sources. The duality only applies to mass and spin, both of which are particle labels and black hole labels, strictly because the Poincare Gauge Theory has the same local symmetries of Minkowski spacetime, yet also reduces to vacuum General Relativity, when all sources are removed completely.

Particles become extended geometric objects when one treats spin on equal footing to mass. The ring singularity does indeed have a principal null direction and newman-penrose tetrads confirm that the ingoing and outgoing vectors are complex-translated off the light cone. However a cartesian Kerr-Schild tetrad can be chosen that is globally Lorentz invariant(see the python test suite below), which implies

gravitational propagation at exactly the retarded time of a worldline. This means that the self-energy of a black hole is removed completely, because individually, black holes(including rotating ones), propagate freely on a minkowski spacetime background. So for a single dirac fermion in the kerr-schild background, the wavepacket propagates on the minkowski background(which treats the light cone as originating from a moving point source), and the gravitational field it produces has a ring singularity that all other wave-packets and all other black holes will experience. This means that although the gravitational field can include a ring singularity, it is an extended object that maps cleanly to a particle worldline in flat minkowski spacetime.

Enter the cartesian Kerr Schild spacetime geometry! In laymans terms, it is the spacetime geometry for a rotating black hole in orthogonal light cone coordinates. They have some nice properties, such as being exactly linear solutions to vacuum general relativity. They have constant metric determinant, are lorentz transformed solutions of one another, and in these coordinates, the vacuum einstein's equations, becomes a simple tensor relativistic wave equation. Everything is exactly linear! No gravitons, no renormalization, no issues, just solutions that coorespond to point particles. And since, these solutions are the result of an exactly linear coordinate system, tells us that the superposition principle still holds. Therefore, one can reconstruct the entire spacetime geometry in the Kerr-Schild form.

We need to propagate the standard model as a quantum field theory over this curved background geometry. We introduce covariant derivatives for all fields. However for the matter fields which use Dirac spinors, there is a sublety, which needs to be addressed: We need the spacetime frame instead of the metric tensor. Luckily Kerr himself, used the so called tetrad approach to produce his spacetime geometry for a rotating black hole. In some research articles, it has been reproduced in cartesian Kerr-Schild coordinates. Using a set of light cone adapted null geodesics, one can produce the correct tetrad frame field.

It is the cartesian Kerr-Schild spacetime geometry, that allows one to treat black holes as particles. Traditionally, one uses curved coordinates of spacetime itself. Spacetime becomes the gravitational field, and the equations become hard to solve, precisely because gravity sources gravity. Since one uses curved coordinates to start with, it is not to obvious to tell which are more useful than other coordinates.

As it turns out, the cartesian Kerr-Schild coordinates are uniquely special, in that they separate the gravitational field from the spacetime background. What's more amazing is that this spacetime background is the Minkowski spacetime from special relativity, which means that the gravitational field evolves as a non interacting, massless, spin 2, relativistic wave equation. We can now apply many new ideas from electromagnetism to gravity, like the retarded potentials, null hypersurfaces, and so on, and yet, also be able to understand why it was the coordinate choice, not the linearity, that explains why general relativity is so highly nonlinear.

The main culprit to why gravity becomes self interacting, is a non kerr-schild coordinate choice. Think of it, if one chose spherical coordinates in say the maxwell's equations of electromagnetic theory, one would need to use much of general relativity anyways to write this spin 1 wave equation in these annoying coordinates. The same thing applies to the Kerr-Schild gauge.

In this coordinate system, gravity is a spin 2 field equation, and like electromagnetism, which is spin 1, changing coordinates results in non linear couplings. In fact this is why self interactions seem to turn off in this coordinate system. Even more amazing than this, is that the inertial frames in spacetime become globally invariant to Lorentz transformations, just like in special relativity.

In general relativity, it only works per point, or locally. By using full blown General Relativity, one looses this important distiction, and the curved spacetime background causes confusion. In a sense, general relativity, only actually becomes necessary for curved backgrounds, as we saw for Maxwell's equations in a spherical background. However by extracting out the Kerr-Schild form, one can indeed show that the gravitational field and the spacetime background are indeed separate.

One might wonder if elementary particles of the standard model are sub-planckian, then what happens to hawking radiation? Hawking radiation assumes that quantum field theory in curved spacetime is fundamental, giving rise to the decay. The decay ceases to occur in classical general relativity, so by treating black holes as stable particles, with all fields classical(non-quantum), we avoid the decay, and thus the duality holds.

So far, we know that general relativity has both local and global symmetries which are only space-time symmetries, making it ideal to map particle states to black hole solutions. Now for the rest of the book, we will asssume the duality as exactly true, and see where this leads us. With the equations we built in cartesian Kerr-Schild coordinates, we note that the kerr-schild tetrad used for the dirac equation in its own kerr spacetime already assumed this coordinate system for the duality. Therefore we will also establish the cartesian Kerr-Schild coordinate system as the coordinates used in the duality mapping. From here on, things become fairly straight-forward, and we lay out hope that this black hole particle duality will survive all of the tests of modern physics, given what we know about scale boundaries and CGH physics.

In the duality, the mass of a black hole M is given by its own particle mass m.

$$M_{BH} = m$$

```
M_BH = m
```

Also in the duality, the angular momentum of the black hole a, is given by its own particle's intrinsic spin j.

$$a_{BH} = \frac{j \hbar}{m c}$$

```
a_BH = { frac{j hbar}{ m c} }
```

For electrons of the electronic field, both the black hole mass and particle mass equal to the electron mass.

$$M_e = m_e = 0.51099895069$$

```
M_e = m_e = 0.51099895069
```

Also for electrons, both the black hole spin and the particle spin equal to the electron's spin.

$$a_e = \frac{j_e \hbar}{m_e c} = \frac{\hbar}{2 m_e c}$$

```
a_e = { frac{j_e hbar}{m_e c} } = { frac{hbar}{2 m_e c} }
```

For photons of the optic field, both the black hole mass and particle mass are set to zero, since photons are massless.

$$M_\gamma = m_\gamma = 0$$

```
M_%gamma = m_%gamma = 0
```

Also for photons, both the black hole spin and particle spin equal to the photon's spin, which in the limit where mass goes to zero becomes the helicity.

$$a_\gamma = \frac{j_\gamma \hbar}{m_\gamma c} = \frac{\hbar}{m_\gamma c}$$

```
a_%gamma = { frac{j_%gamma hbar}{m_%gamma c} } = { frac{hbar}{m_%gamma c} }
```

What makes this interesting is that mass and spin are the only parameters of the Kerr solution in cartesian Kerr-Schild coordinates, and are also the same parameters that relate to space-time symmetries that motivated our decision to keep the duality. So far so good, the symmetries allow for mass and spin to be equated which is consistent with the duality. Now its just a matter of including the other symmetries like electric charge and what not, that don't participate in the duality, which means the duality doesn't couple to charges, and we can set the charges to be what we want. This allows for a huge freedom in our understanding of particle physics. The standard model has lots of internal symmetries, and it has agreed to every experiement ever done. So clearly I am not saying that electric charge doesn't exist. The duality allows such charges to exist, its just that they won't couple to the spacetime geometry of gravity. That is another good sign, that this duality is on the right track.

For the duality to work, it has to explain what general relativity and quantum field theory fail to explain. The standard models of particle physics and cosmology have gone far beyond anyones expectation, and this is to be taken at face value. Before we move on, here are some of the solutions that the duality has to the current unsolved problems of physics.

84% of matter sourcing gravity is unknown. The other 16% is known to obey the standard model of particle physics, and agrees with experiment without any deviation. Given that it doesn't obey the standard model, the assumption goes that one must introduce new degrees of freedom to the standard model that only interact via gravity and possibly extremely weakly via the known standard model particles. But experiments searching for such particles have all come up empty handed.

The standard model of particle physics has produced total agreement with experiment. Taking this at face value, one must confront the possible reality that the 16% of known matter is all there is to the standard model, as anything beyond the standard model is speculative as a dark matter canidate. If one assumes my hypothesis as true, then in general relativity, black hole singularities go from being undefined geometries to being particle worldlines. Given the classical geometric particle interpretation of black holes in the kerr-schild formulation of general relativity, one has a perfect dark matter canidate for particle dark matter. Black holes have mass and spin, just like wigner's classification of casimir invariants but are not needed to obey the constraints given by the particles of the standard model. As such, the 84% of dark matter is likely black holes as this becomes the only remaining option due to the standard model as dark matter being ruled out. But this is only possible if black holes are particles.

The current consensus is that there is a single observable universe that obeys a single law of physics. It would be nice and elegant if the standard model could be explained via a unifying principle. String theory does exactly this, as it posits that all the standard model particles are different vibrational modes of the same string. Problems arise when one demands consistency with reality. The theory predicts an 11 dimensional spacetime known as membrane theory, and it is not known which of the 5 different dualities of supersymmetric string theories with their ~10^500 different compatifications gives our reality. Such a waste of physical theory beyond reality seems unlikely given Occam's razor, which states that the simplest option is the likely one, given equal likelyhoods for each. The problem is that the standard model with all its variety more easily fits the bill than a single type and single compatification of string theory.

If the duality between black holes and particle holds up, equating the mass and spin of all the standard model particles with the mass and spin of sub-planckian rotating black holes, implies that black holes are the unifying thread, not strings. As said earlier, string theory would also be a unification option, but with such complexity beyond our current reach of experiement. For example, the failed detection of super-symmetric particles, dark matter particles, new short range forces, proton decay, etc, only pushes the theory back further. The black hole particle duality requires no extra particles or dimensions that string theory assumes, except for a single assumption that spacetime geometry is of the kerr-schild class. And moreover, the duality rests only on the solutions of general relativity which has also passed every experiment to date. Given that both strings and black holes solve the unification problem, and that black holes are confirmed reality within a tested general relativistic framework, but strings are not, gives us enormous promise given to us by Occam's razor, which would prefer black holes over strings as the unifying principle. And this would unify not just the standard model

but every particle as a type of black hole solution, including new particles yet to be discovered.

In order for general relativity to be compatible with quantum field theory, which is the framework in which the standard model rests, is a huge undertaking. One quantizes the field theory to arrive at a particle interpretation state of excited field modes. If one did this to a geometric theory like that of general relativity or poincare gauge theory gravity, one ends up with non-renormalizable graviton vertices and certain particle states that are unbounded from below. Trying to fit a geometric field theory within quantum mechanics is an unsolved problem. The evidence against quantizing spacetime itself is mounting. String theory and loop quantum gravity among other approaches are baroque in their complexity and require more assumptions than usually permitted. Given that track record is so poor without experimental evidence in any direction is telling that quantum gravity is likely impossible if not completely intractible as it stands. This is a lot of counter evidence. So given that gravity must be quantum, and every approach to it leading to failure, should one stop trying, and look for loopholes in the argument. The problem is that a solution might require sidesteping the problem entirely, but nobody to date has shown a counter-example. This is the problem.

In quantum gravity, one assumes that one must quantize the metric or in loop quantum gravity one quantizes the connection. But under the assumption that the black hole duality is true, it allows one to use the kerr-schild geometry as a space-time background, General Relativiy becomes exactly linear and all the higher order graviton terms vanish. Also given linearity one can superimpose certain solutions. So as long as one stays within the kerr-schild coordinate system, one could avoid quantization of spacetime, and only quantize the black hole singularities. This quantization of particle worldlines in a minkowski background that the duality allows, gives us the relativistic wave equations that any free particle obeys. Therefore a particle's mass and spin would determine the black hole geometry from the get-go, without needing to quantize the spacetime metric or connection itself. This implies that gravity is the superposition of spacetime geometries from the mass of each black hole, and torsion is also arising out of the spin of each black hole. Since super-positions are only allowed in a certain lorentz frame when the worldline singularities are geodesic, and since all relativistic wave equations are geodesic in the classical limit, tells us that carefully integrating lorentz boosted black hole geometries in the kerr-schild frame over the wavefunction of the universe, would give one the spacetime geomety and as such solve quantum gravity without quantizing space-time itself.

In quantum mechanics, there is a wavefunction that evolves via the shrodinger equation. It doesn't matter if one is quantizing particles or fields or even information, the wavefunction is real and determines measurement outcomes. The problem is the following notion that the wavefunction must collapse to a definite outcome upon observation and there is no consensus on what such an observation entails.

In quantum field theory which quantizes fields such that of the fields of the standard model, the wavefunction of the system contains many particle states and their transition amplitudes, leading to a debate of ontological beables. However, as we noted in the black hole duality, the gravitational field is classical, and the only thing that is quantized is the particle interpretation itself. Assuming the duality as true, would mean treating all fields as classical. As such, particles are not fundamental as they are classical wavepackets within a classical relativistic field. In simulations of any relativistic wave equation, multiple wavepackets can traverse independently, just like multiple identical particles. The shrodinger equation for a single particle becomes the non relativistic limit of the classical field. And without wavefunction collapse, the wavepackets would not be zeroed out to a single location or frequency. The entanglement of a many particle state would be traceable to starting with a set of classical particles and then transitioning to the quantum theory. In a classical relativistic field interpretation, the field becomes the wavefunction directly, already containing a quantum particle interpetation.

Section 5.4: Solving a Multiple Black Hole System

For a single particle which propagates on a minkowski background, naively summing the metric will break linearity. This is due to the tetrad(I mention this in my python test suite too), which would have to be passively transformed, misaligning the principal null-direction. However, even in the curved spacetime geometry of many other black holes in the kerr-schild formulation of general relativity, the superposition principle will continue to hold. This is because one actively(but not passively) transforms the retarded time effects(the past lightcone off the point-like worldline) to its rest frame, and computes the geometry there, then sums the geodesic accelerations and geodesic spin-transports, and not the spacetime metric. It is this, that circumvents the issues that break linearity. And it all holds because the tetrad I use is globally Lorentz covariant, allowing lorentz transforms to the rest frame and as such can simultaniously have both an accelerating rest frame which is actively transformed, which keeps the null vectors aligned, which is passive. If one actually tried to passively boost the tetrad rest frame away from its natural geodesic(free-fall) motion, yes the linearity will break once more. This is because(like we stated earlier in the discussion on super-extremal spin issues), the gravitational field is seperate from the flat minkowski space-time background. So while the black holes seem to accelerate in the minkowski spacetime background, they are still following geodesics in each-others gravitational fields. It is geodesics that allow superpositions, not deviations from them. It's just that for a single particle the geodesic becomes a inertially straight line, giving us the standard result.

My python test suite builds the tetrad that I guessed at, and runs it through a battery of tests. The tests were extensive and their sole purpose was to falsify the tetrad and its relation to the 1975 paper by Gurses and Gursey, called the Lorentz

Covariant Treatment of the Kerr Schild Geometry. I don't trust anything online, even with AI running everything. So to show that the paper had a sound argument, I decided on a battery of tests that the paper just assumes.

These included the determinant of the metric, which tells you about the 4D volume of spacetime, and how it changes per point. The paper claims that the volume is the same everywhere. But if the volume changes, it invalidates the whole paper, and my simulation would be back to square one.

Besides the paper, I also had to check if the tetrad I guessed at, works to build the kerr metric in cartesian kerr-schild coordinates. Since we have a tetrad, checking to see if the tetrad recovers the flat minkowski spacetime is a powerful consistency check, that any tetrad should have. Then because we have tetrads now, we can go back to our volume test and find out whether or not the determinant works for the tetrad as well as the metric.

So the paper makes a claim about volumes, my tetrad was a complete guess, and of course my black hole particle duality is a thing that has to work if I want to build a game out of this. For this I looked to quantum field theory and particle physics, specifically for the plane wave solutions to the dirac equation. I understood immediately that the mass and spin, of the black hole have to match the mass and spin of the particle, but there is no guarantee that this is satisfied.

So I built the lorentz tetrad test. I generalized my initial guess to include an arbitrary lorentz frame, such that the tetrad tests should pick up on any nonlinear effects that the rest frame might introduce to other frames. As a reminder spherical coordinates are like this, so if I somehow was developing a null Newmann-Penrose tetrad using spherical coordinates, a change in tetrad by lorentz transformation will mess up the tetrad at other points. In other words, tetrads are only transformed locally, not globally. So then, this lorentz transformed tetrad test does something new, it computes the rest frame tetrad, computes active and passive lorentz transforms at differing points, and then compares them.

If this passes, we have lorentz transformed the tetrad consistently, and if the other tetrad tests pass, we have recovered the kerr metric for a rotating black hole. If both tests pass, we have tetrad that represents a particle. Then alongside the 1975 Kerr-Schild paper, I could show that my tetrad, which you remember I only guessed at in 2009, should pass all the tests. If so, it grants the ability to build a black hole orbiter.

Then… Every test passes to machine precision!!! With a working kerr-schild tetrad, I could finally use it to build my black hole orbiter. As I said earlier, this code is an exact reformulation of numerical general relativity. You see, most, if not all GR codes, just solve the Einstein Field Equations directly by brute forcing the solution. As one would expect this requires supercomputers and is a slow procedure.

What I discovered, is that it is possible to exactly solve general relativity, so as long one stays within the kerr-schild coordinate system. To do this requires one to accept my black hole duality as true, which is a heavy assumption for many scientists to handle. Most relativists wouldn't consider black holes to be point like geodesics. But I did, and because I saw reality in this way led me to write this code that according to Occam's razor, says that the simplest solution to the same problem is likely the correct solution.

My black hole orbiter code unifies several key concepts:

1). The minkowski background spacetime of special relativity, that keeps light speed constant for all observers.

2). Newton's laws of universal gravitation, for the self attraction of gravity, which follows an inverse square law.

3). Many particles are involved, each with their own conserved properties, such as mass and spin.

Through this unification, one must realize that a rotating black hole is a particle, and the only way to have a particle simulation of multiple rotating black holes is to use the kerr-schild tetrad that I discovered to be correct. In so doing the black hole orbiter code has to tell apart retarded time, coordinate time, and proper time, all which play a role in the simulation.

To obey special relativity, the gravitational influence of one black hole around another can only propagate at the speed of light. Otherwise the simulation would be frame dependent, and be broken. This is because the speed of light is a constant in special relativity. And in the kerr-schild coordinates, one still has a minkowski background, which means that whatever nonlinear effects might appear in the kerr-schild tetrad for the gravitational field of a source, its effects will always still propagate at the speed of light in a minkowski background.

This effect is known as retardation, and the time it takes for a signal from a gravitating body to reach us is known as its retarded time. In otherwords its a direct relationship between distance and retarded time. The futher one looks, the more time it takes for its light or gravity in this case, to reach us. For example, in astronomy, when I asked that simple question of why the universe ends at 26 powers of ten, and why a 13.8 billion year old universe is possible, is all because of retarded time.

The other two types of time are proper time and coordinate time. Telling them apart is simple. Coordinate time is the time we all agree on, just as one agrees on the defintions for any set of coordinates. If I arbitrarily chose a set of coordinates, say cartesian coordinates X, Y, and Z for space. The fourth dimension T in that list would be coordinate time.

However, your personal time slows down in this 4 dimensional spacetime, when moving at high speed, close to light speed. Proper time is personal, and depends on how

fast you are moving relative to your coordinate grid. That is how in 1 tick of proper time, the time you experience, coordinate time passes faster by any amount of time greater than 1 tick of coordinate time. That is how if one moves very close to light speed, one can in effect, travel into the deep future, because one's coordinate time would have passed by an enourmous amount from somebody else's perspective.

So put together, retarded time is calculated from the past history of all other particles, using a binary search algorithm, to find the exact moment when the source crosses the past light cone, of the particle in consideration. Then every particle propagates forward through its own proper time, up to a common coordinate time for consistency.

Then we get to the problem of multiple gravitating sources. To do this properly, one must sum over the accelerations, and not simple add the geometries, which is impossible, even for kerr-schild coordinates. And it only works if one lorentz transforms to the frame in which the source is moving. Therefore, the principal null congruence, or the "L" 4-vector, stays oriented correctly within spacetime.

For an observer, one must choose a given black hole as the observer, and render to the screen the retarded positions of all the other black holes in the frame of that observer. This means that the targeted coordinate time depends only on the coordinate time of the observer, which in turn depends on the observer's own proper time. In other words, the time you experience as an observer outside of the game, is the same proper time that passes in game for that given observer.

With all of these frame by frame lorentz transformations, to calculate accelerations, find the view from an observer, using retarded time lookups, proper time evolution, and coordinate time synchronisation, one has everything one needs to take the kerr-schild tetrad I confirmed works, and build it into a working black hole orbiter.

My code(same as before), is put together correctly, and works correctly. It includes my lorentz covariant kerr-schild tetrad, following the 1975 paper, and using my test suite, which passes to machine precision. Lorentz transforming to the source and accumulating accelerations in each particle's proper time, consistent with the Einstein field equations being linear and superimposable in the kerr-schild gauge, means the code as written is correct.

I tested it against inspiral vs central force and saw that it only inspirals when the test mass is significant. I also tested the spin orbit coupling via frame dragging in the rotation sandwhich, and it is consistent under a, x to y to z to x swap. It also works for three or more(after I found out how to print everything) particles at once, and also the proper time sync works too because the coordinate time for ultra-relativistic particles wasn't perfectly rounded, which means that fewer proper time ticks passed in that interval, as expected. Everything is working.

Now that I got it working, its time for me to integrate a visualization layer. Something that takes these datapoints and plots them in the... Wait!!! Black hole particle

duality isn't enough. Every particle is a black hole, wigner rep, collapsed classical field wavepacket, and hear this: An observer. The singularity didnt stop existing at the big bang. We are the singularity interacting with itself. If true, then my code doesn't need a renderer, as it already is a renderer. The so called "graviton's" reaching a black hole are like the photons collapsing in my retina. So a retarded time snapshot in the player frame isn't required. Its built in because, every observer is a black hole.

Here is the full tetrad used with arbitary lorentz transforms. This is useful for aligning the principal null directions from multiple sources. This tetrad is globally lorentz covariant not just locally like last time. Also the retardation in the gravitational source is also clear.

$$e^{\mu}_{A}(M_{other}, a_{other}, X^{\mu}) = \Lambda^{\mu}_{A} + \left[\frac{H}{2}\left(\Lambda^{\mu}_{B}\eta^{CB}l_{C}\right)l_{A}\right]\left[\delta\left(\eta_{\mu\nu}X^{\mu}X^{\nu}\right)\theta\left(X^{t}\right)\right](M_{other}, a_{other}, X^{\mu})$$

```
{e^%mu_A (M_other, a_other, X^%mu)  = {%LAMBDA^%mu_A + left[ {frac{H}{2}
left({%LAMBDA^%mu_B %eta^{CB} l_C}right)l_A } right] left[ {%delta
left( %eta_{%mu%nu} X^%mu X^%nu right) %theta  ( X^t)} right]}} (M_other,
a_other, X^%mu)
```

To find out where all the sources are for a given particle, we need to parameterize a path in relation to said source.

$$X^{\mu}(s) = [x^{\mu}_{BH} - x^{\mu}_{other}](s)$$

```
X^%mu (s) = [ x_BH^%mu - x_other^%mu]( s )
```

Then here is the equation of motion for a multiple black hole system. Something the Kerr-Schild tetrad exactly allows. In fact we do actually sum over accelerations in their own aligned rest frame to obtain a geodesic equation for each black hole particle.

$$\partial^{2}_{s}x^{\mu}_{BH} = \sum_{other \neq BH} \hat{O}_{other}\left([\Lambda^{A}_{\mu}]_{BH}\, e^{\mu}_{A}\right)_{other}$$

```
partial_s^2 x_BH^%mu = sum from {other <> BH} {hat O}_other
( [%LAMBDA_%mu^A]_{BH} e^%mu_A )_other
```

What catches my attention here, is that the black hole particle duality is not assumed, as it emerges naturally from the constraint that each test particle is also a black hole source. If not, we would not be able to recover general relativity. Therefore the black hole duality is not just speculation. It is grounded in how the kerr-schild formalism for General Relativity works. Not many know about this result, but it works in code(see below). Now that we can solve a multi-particle system in general relativity, something Numerical Relativity theoriests say cannot be done properly because GR is non-linear, non-perturbative, and non-renormalizable, blah blah blah… Actually Kerr-Schild coordinates linearize the equations, allow for specialized superposition like we do here, and from this the black hole-particle duality emerges on its own. It only seems to look like an assumption from the outside, but when one studies this alternative in its own right is makes things much simpler. It allows simulations of numerical GR on consumer hardware without brute forcing it. It allows the completion of the CGH physics cube as we have seen. And soon, we will know how to build a theory of everything on it.

Section 5.5: The Wave-Function for Many Identical Particles

Since we have solved a many particle system for General Relativity. Now its time to do the same for Quantum Field Theory! We know that GR is a field theory, but so is QFT. This means that like before, we have to find the particle solutions in the field theory. For GR this was trivial because of kerr-schild coordinates. But, the details matter here, such as why the schrodinger equation has so many people baffled about what a particle is.

For quantum physics 101, we recall how to apply quantum physics to a point particle. Back in classical physics, we know a particle is an actual point that travels in a straight line with a constant speed. Just like a billard ball. It has linear momentum, and kinetic energy. The momentum is just its mass times its velocity vector. Whereas the kinetic energy is simply half of the dot product between its velocity and its momentum. However, the shrodinger equation tells the same story, with a central modification. The solutions of the equation are now sine waves, with their wavelength inversely proportional to their momentum, and their frequency directly proportional to their kinetic energy. By adding many sine waves together, one can create wave packets: Traveling waves that look like a collection of point particles from a great distance.

However, most scientists didn't see it that way. They invented a slew of new methods to coax particles out of the wave equation. Lets examine each case, one by one. So here we return to our original question: What is a particle?

One way is to reinterpret the wave equation as describing not a field, but a probability wave, known as a wavefunction. The amplitude squared of the wavefunction gives the probability density of finding the particle in space. Therefore during measurements, the wavefunction "collapses" to a single point of interaction. This collapse gives us the definite position for our point particle. But this gives new problems. Since the collapse is a result of a single measurement of a single particle, means one thing: One must introduce new dimensions of space to handle more particles… 1 particle is 3D, 2 particles is 6D, 3 particles is 9D, 4 particles is 12D, and so on… This phenomenon is known as quantum entanglement, or spooky action at a distance. And this is used to compute corelations between multiple particle probabilities.

Another way is to keep our original interpretation of the wave equation as describing a field. From this, we apply the whole procedure of quantum physics all over again, not to our point particle, but to the entire field! The "quantum" field as it is called, has an infinite number of dimensions, each one describing the value of the field at a particular point in space. Each sine wave in the field, has a physical amplitude, just like the position of a particle. Therefore, the resulting "quantum field" is an infinite collection of coupled quantum harmonic oscillators, and is no different than our original use of wavefunctions for describing the system. Then, the energy levels of each quantum harmonic oscilator, corespond to a collection of identical quantum particles. From this, we have come full circle, as "quantizing" the field, gives the same result as

reinterpreting the field as a multiple particle wavefunction. Something seems off here, due to the persistence of point particles.

The reason for using quantum mechanics then has nothing to do with the shrodinger equation itself, and rather is all about the probabilities of measurements having to do with some sort of wavefunction, which lives in an abstract space of possibilities known as a hilbert space. What we have done is separate the quantum stats from a given classical field. Akin to seperating the background spacetime from the gravitational field of a rotating black hole. Its the seperation of classical relativistic fields from their quantum or gravitational counterparts that allows unification of mutliple particles at once! This is why it is possible to start with nothing: The void, look at its symmetries, and define all of physics from it. We have in a sense everything from nothing. A Theory of Everything!!!

To get there, imagine one has a plane wave solution to a relativisitic wave equation, like the dirac equation or proca equation we defined earlier in the book. Setting the dirac mass to be that of the electron gives us an operator we can use for the electron. Photons are less forgiving, as we have to set the proca mass all the way to zero, and this recovers Maxwell's equations for electricity, magnetism, and light. Maxwell's equations in their 4 vector form are used as another operator, that this time, we can use for a photon.

Here is an operator that we act on the wavefunction for a single electron. We defined its coupling to the black hole particle duality above.

$$\hat{O}_e \Psi_e = 0$$

```
{hat O}_e %PSI_e = 0
```

Here is another operator that we act on the wavefunction for a single photon. Like before we defined its black hole coupling too.

$$\hat{O}_\gamma \Psi_\gamma = 0$$

```
{hat O}_%gamma %PSI_%gamma = 0
```

These two operators, now become partial differential equations that act on their single particle wavefunctions. Their solutions for the non-interacting case we are talking about here are all klein gordon plane waves. Therefore, each plane wave carries a definite linear momentum 4-vector. When one superposes many plane waves together, one can construct wavepackets that traverse and spread out as they travel. This is the uncertainty principle in action.

It is interesting to note that this is nothing more than a Fourier analysis. A single particle wavefunction can be fourier transformed to find the frequencies that make it up. Since each frequency is a plane wave, means one thing, that when we look at a wavefunction for many particles, the equations they obey will be the same, because the plane waves each dimension decomposes into, are also the same… Here is such a multiple particle wavefunction:

$$\Psi_{e_{N_e}\gamma_{N_\gamma}} = \prod_{n_e=1}^{n_e=N_e} \prod_{n_\gamma=1}^{n_\gamma=N_\gamma} \Psi_{e_{n_e}} \Psi_{\gamma_{n_\gamma}}$$

```
%PSI_{e_{N_e}%gamma_{N_%gamma}} = Prod from n_e=1 to n_e={N_e} Prod from
n_%gamma=1 to n_%gamma={N_%gamma} %PSI_{e_{n_e}} %PSI_{%gamma_{n_{%gamma}}}
```

As soon as one has multiple particles in the wavefunction, the dimensions of the wavefunction explodes. Since the equations are relativistic, we have a many-time, many-particle wavefunction. In fact we have a single wavefunction with 4N dimensions. N time dimensions and 3N space dimensions. In other words, such a wavefunction contains an entire Minkowski spacetime branch per particle.

As we can see, its just a huge Kronecker product of single particle wavefunctions. So if we have particle 1 here and particle 2 there, there exists a wavefunction where if we project down the wavefunction to a given particle, we can extract the positions. That is why it has to have many space-time copies. Each background Minkowski space-time, is a particle branch of the wave-function.

Now we build our many particle states. For a single particle at a time, like a single electron and no photon, or a single photon but no electron, we have the following single particle state.

$\Psi_1 = \Psi_e(x_1^\mu) + \Psi_\gamma(x_1^\mu)$

```
%PSI_1 = %PSI_e  (x_1^%mu ) + %PSI_%gamma  (x_1^%mu )
```

Now, onto the two particle state, it is more complicated… Why so? Since every electron is identical to every other electron, and every photon is identical to every other photon, we cannot tell them apart. We don't know if the plane wave with 4-momentum index of 1 is for the first or second particle. Because we cannot tell them apart. The same is true for the 4-momentum with index of 2. Therefore, one has to superimpose solutions that keep the same momentum, but swap the positions. One can get the exact same result if one keeps the same positions, but swaps the momenta. Since our single particle wave-functions are built from plane wave solutions which have definite momenta and undefined positions, we will superimpose solutions that keep the same momentum, but swap the positions.

Since this now works, one must ask if one should add the 2 particle wavefunctions or subtract them. For electrons one subtracts, but for photons one adds. Why? Half-integral spin fields like Dirac fields are first order in their space-time derivatives, while integer spin fields like Maxwell fields are second order in their space-time derivatives. The second order nature like the Box/Laplace operator is commuting because the derivative stencil on spacetime is symmetric upon exchange. Likewise the first order nature of the Gamma-matrix derivatives acting on the classical spinor fields for the dirac fields for example, is anticommuting(one can check that the matrices anti-commute into the metric tensor), which means that there is no need for grassmann valued fields as the spinors themselves already anticommute. The answer to this is because the derivative is first order in spacetime and is anti-symmetric upon exchange. The thermal nature of the fields thus arises from perturbations(weak and strong) that causes entropy production, like a billiard ball in a pool table, would eventually reach equilibrium. This equilibrium will obey the right statistics because the wavepackets would

interfere thermally(akin to perlin noise vs a single coherent wavepacket spreading), and since these wavepackets have a energy dimension that depends on the space-time derivatives, their overlaps will also be dependent on both the temperature(which is inversely related to the microscopic resolving distance), and the spin-content, as we have seen is also given by the space-time exchanges of derivative order(note we discuss this too in the quantum vacuum issue).

So we have five terms to superimpose together into the 2 particle state. As we can see in the last four terms, we have two electrons, their swapped versions, then two photons, and their own swapped versions as well. One can see that when the position 4-vectors for particle 1 and particle 2 are swapped, the two electron case gets subtracted from, but the two photon case gets added on. The first term of the 5 terms is simpler, as it involves two particles, but we can tell them apart, so no switching is needed, as one electron and one photon have differing mass and spin. Yet they make a two particle state, so we include them. Here are the five terms for the two particle state.

$$\Psi_2=\Psi_{e,\gamma}(x_1^\mu,x_2^\mu)+\Psi_{e,e}(x_1^\mu,x_2^\mu)-\Psi_{e,e}(x_2^\mu,x_1^\mu)+\Psi_{\gamma,\gamma}(x_1^\mu,x_2^\mu)+\Psi_{\gamma,\gamma}(x_2^\mu,x_1^\mu)$$

```
%PSI_2 =  %PSI_{e ,%gamma} ( x_1^%mu, x_2^%mu ) + %PSI_{e,e} ( x_1^%mu, x_2^%mu ) - %PSI_{e,e} ( x_2^%mu, x_1^%mu ) + %PSI_{%gamma,%gamma} ( x_1^%mu, x_2^%mu ) + %PSI_{%gamma,%gamma} ( x_2^%mu, x_1^%mu )
```

We could go on, and include a 3 particle state for electrons and photons, but that would get very unwieldy and the number of terms would explode out the window. For a taste of this. If one must find all the permutations between three electrons for example, we have 6 options, and all even permutations of their 4-vector positions are all positive and added on, wheras all the odd permutations of their 4-vector postions are all negative and subtracted from. Then, if one must find all the permutations between three photons, we do the same procedure, but keep all 6 options all positive and add them all together. We call such a procedure symmetrization for photons, and anti-symmetrization for electrons.

Luckily for such high N situations there are functions used for this. If one has 4 identical particles one would have 24 options. For 5 one has 120 options. And so on, going as N factorial. One could use a matrix between all 5 positions and each of their 5 momenta. By taking the determinant of this matrix, one would get the exact same answer for electrons. But for photons, permanents are used instead.

For situations where one has 3 electrons and 2 photons for example in such a 5 particle state. One takes the determinant of the 3 by 3 electron matrix and the permenant of the 2 by 2 photon matrix first, then mutiply the two answers together without any swapping, because we know the difference between electrons and photons. The same goes for all other combinations of electrons and photons that adds up to 3,4,5,6 and so on total particles.

Lastly one just superimposes all these many particle states together. We call such a state a fock space, which is a kind of hilbert space that is identical to a quantum wavefunction for a field theory, not a particle theory.

$$\Psi = \sum_{N=0}^{\infty} \Psi_N$$

```
%PSI = {sum from N=0 to infinity  %PSI_N}
```

We should pay attention to what the schrodinger equation is actually describing, and not shove particles into our paradigm. By doing so, we lose track of what reality is, as human centric constructs enter our most fundamental theories. Particles are just one example of this. In everyday life, we see solid objects in motion: Particles. And objects move through a physical medium: Space. If particles are such a human centric construct, we have to make sure that this doesn't creep into physical theory…

However, there is more to the shrodinger equation than this! Every relativistic wave equation, reduces to the shrodinger equation in the domain that the speed of light becomes instantanious. So, if the shrodinger equation describes a field, and this field is what one gets in this regime where the speed of light tends to infinity, means one thing: All relativistic wave equations describe fields too!

As it turns out, this is all consistent will Bell's inequality violations, because fields are used. You see, the measurement problem goes away when one recogizes that "quantum effects" emerge from classical fields and not canonical quantization. The confusing detour that field quantization does, is it allows for a fock space factorization, giving spin stats and entanglement. However the single particle state obeys the same free field theory(only with the wavefunction interpretation intact) as the classical field one started with. Classical fields already display uncertainty(through fourier spreading), quantization(due to possible non-linear couplings to other fields and black holes), superposition(because multiple wavepackets exist) and entanglement(because these wavepackets propagate independently). Bells theorem only bites when one accepts the field as a wavefunction that collapses to a measurement point in a Hilbert space with many space-time branches. The objection is a category error in the simplest form, and since fields are non-local in the sense of being spread out, in the first quantized sense, this produces bell's violations exactly as expected. But in the second quantized sense there is no bells theorem to be worried about because fields not particles are what exists. After all, we know that classical fields give rise to quantum phenomena, because of the use of a pure field ontology. With a fields-only perspective, vacuum flucuations do not exist fundamentally because of the operator quantization of the fields, but already emerge from the classical fields due to the fact that fields are spread out entities. I mean to say it's not like one is tracking any bohmian paths within it whatsoever. Because of this, lennard jones type behavior still occurs naturally, and happens because the fields are locally differentiated contiguous entities, not sharp localized points. Even in the case of the black holes, one still integrates over the fields, albeit carefully.

Section 5.6: Including Interactions that Change Particle Number

As it turns out, a wave can only affect other waves nearby. This means that it takes some time for a wave over here, to influence a wave over there. The sphere of influence grows at a constant speed. This speed is the speed of light. Since light speed is always the same, means that from a moving perspective, it is time itself that runs slow. As such, space and time are unified into a new framework. Spacetime! This is how special relativity enters our understanding of the universe.

Imagine finding a concentrated lump, or packet of waves. Seen from far away, this so called wave packet, looks very much like a dot, moving in a straight line with a given speed. A particle. However, "particles" act very strangely at microscopic distances. They act randomly, yet also are in many places at once. How can this be? This is because they are not particles, but waves. After all, waves are spread out entities. So it makes sense why particles seem to act like this. This wave like nature is in the foundation of quantum physics.

In what follows, almost all everyday phenomena, from that of chemical reaction rates, to the way viscous fluids flow, will be determined by how two types of waves interact. They are the electronic waves and the optic waves. The optic waves and electronic waves can only support a discrete number of quanta. Any wave is a superposition of plane waves, and the amplitude of each plane wave is directly proportional to the square root of the the number of quanta residing in it. Electrons of electricity are the quanta of the electronic field and can hold only binary numbers of quanta. Photons of light are the quanta of the optic field and can hold any whole number of quanta.

So, given this, the electronic waves and the optic waves can act very differently. Electronic waves, have a property called mass, whereas optic waves do not. All waves with shorter wavelengths, tend to vibrate with a faster rate, or frequency. It is this excess frequency, in addition to its frequency due to its wavelength, that is the mass of the waves. There is a formula for this excess hum, and it goes as follows: The mass of a wave squared is directly proportional to a difference of squares, similar to the pythagorean theorem, just backwards. It is the frequency squared minus the magnitude of the wavenumber squared. It is the fact that time squares to a negative number respect to the three dimensions of space, that is the key definition of special relativity. This is the reason why some waves, have the property that they can vibrate faster, yet be very spread out. These waves are our electronic waves.

However, there is more! Mass isn't the only property that waves can have. Waves can also have a property, known as spin. That is to say, that the direction that the waves are vibrating in, its polarization, depends on our point of view. The amount of spin, takes on discrete values, of zero, one half, one, one and a half, two, and so on. For example, we say that, electronic waves are spin half, because it takes two full turns for their polarization vectors to take one full turn. On the other hand, optic waves are spin one, because under a full turn, its polarization vectors also take a full turn. Otherwise put,

the more ways, or degrees of freedom, the greater the spin. As is turns out, in the general case, quanta with odd multiples of spin half all have binary numbers of quanta in each of their plane waves, and quanta with even multiples of spin half all have whole numbers of quanta in each of their plane waves.

Back to electrons and photons, the electronic waves can also experience electromagnetic forces, meaning that they can push and pull on each other. This happens, because electronic waves and optic waves, can interact. If we take two electronic waves, placed some distance apart. They might attract one another, because both electronic waves are disturbing the optic waves in between. This interaction between two electrons and one photon is why we exist, and is characterized by the electric charge.

To see how we get interactions that change particle number, lets take a look at this theory of interacting electrons and photons. The electrons are digital vibrations of the electronic fields and the photons are also digital vibrations, this time in the optic fields. We denote the electronic fields together as a lower-case psi, and the optic fields together as a capital A. Here is the lagrangian for QED, the theory of interacting electronic and optic fields.

$$\mathscr{L}_{QED} = -\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)\left(\partial^\mu A^\nu - \partial^\nu A^\mu\right) + \bar{\psi}_\alpha \left(i \gamma^\mu_{\alpha\beta} \left(\partial_\mu - i A_\mu \sqrt{\frac{4\pi}{137.036}} \right) - \delta_{\alpha\beta} m_e \right) \psi_\beta$$

```
Laplace_QED = -{frac{1}{4}}{left( {partial_%mu A_%nu - partial_%nu A_%mu}
right) left( {partial^%mu A^%nu - partial^%nu A^%mu}  right)} + {bar %psi}
_%alpha left(i %gamma^%mu_{%alpha%beta} left( partial_%mu - i A_%mu
{sqrt{frac{4 %pi}{137.036}}} right)  - %delta_{%alpha%beta} m_e right)
%psi_%beta
```

This lagrangian although a small part of the full theory of everything, is a great starting point. That is because it involves electrons and photons, particles that we encounter in daily life, even to non-physicists. Electrons are reponsible for the structure of matter, including its chemistry, and is directly felt as electricity. Photons are known to us as light, as light itself is nothing more than a stream of photons. Since chemistry is the foundation for biological matter, everything we experience is a large part due to QED. Everything except for nuclear reactions and gravity, which most are unaware, unless you are an actual power plant worker or astronaut, you don't know anyways what's beyond QED: The strange theory of light and matter!

Rememeber the multiple particle wavefunction from last section? The total wavefunction is just a sum over many particle states beginning from 0 particles, the vacuum, to 1 particle, which could be an electron or a photon but no more and no less. Then we have the 2 particle state, where exchanging two photons was symmetric and exchanging two electrons was anti-symmetric. So the 2 particle state was five terms.

The first term was just a product of a single electron state and a single photon state. The other four were as follows, the unswapped pair of electrons and its swapped version, and the unswapped pair of photons with its swapped version too. The symmetry and anti-symmetry is accounted for by labeling another's particle with the current set of

coordinates. If the other particle has another momentum than the first, it appears as if the two particles have either their momenta fixed and the positions swapped, or vise-versa.

Since this swapping occurred between solutions to our single particle wavefunctions, which are detemined by using the appropriate black hole equation from our black hole particle duality, we automatically got out the correct plane wave behavior for all single particle states. The symmetry for photons and anti-symmetry for electrons only happens when one constructs these multiple particle states, ready to be summed up into the full wavefunction, known as a fock space, which is a special kind of hilbert space for quantum field theory.

One can now define the first particle state for an electron, by using the field as an operator that transitions us from the vacuum to a single particle state. Basically we are just taking a huge product of fields. And since fields vibrate at their frequency in any such plane wave, multiplying their waves is equivalent to adding their momenta just as we would expect. 10 identical marbles have 10 times the mass as 1 marble by itself.

For the electron state we can act on the vacuum by the dirac operator of QED.

$\Psi_e = \psi_\alpha \Psi_0$

```
%PSI_e = %psi_%alpha %PSI_0
```

For the photon state, we also act on the vacuum, this time by the maxwell operator of QED.

$\Psi_\gamma = A_\mu \Psi_0$

```
%PSI_%gamma = A_%mu %PSI_0
```

Since both operators move us from the vacuum to 1 particle, we essentially have a way to create a single particle from nothing. Therefore since QED is nonlinear, unlike our previous black hole equations which were just the free or linear parts of the full nonlinear QED where the dirac and maxwell equations interact. The coupling between electronic and optic fields results in a very interesting property. Particle creation and destruction!

Usually this is hidden in plain sight. For a single electron or single photon, travelling freely, not interacting, there are both a psi and psibar for electrons in the lagrangian that says create it, let the dirac equation work on it, then destroy it back to the a vacuum. The same logic goes for the A field for a single photon that gets created to evolve the maxwell's equations then gets destroyed yet again. But these interactions changed that.

Since we are evolving single particle states, which themselves are products of many field operators. We can have the full interacting set of Feynman diagrams produced automatically, that tell the probability of two or more different particle states will interact between differing particle numbers. Essentially this couples together many of our multi-particle states in a way that doesn't use feynman's rules that rely on expanding out in powers of the coupling constant. Since, QCD and QEW of the theory of everything rely on non-perturbative approaches, feynman's rules would be of

less use where we really need it like inside a proton, or bound states like atoms and molecules would be impossible without these many particle couplings within fock space.

Then we can define the operators that act on our single particle states, and since they are non-linear, with many interacting terms. We get terms that change particle number. For example, a single photon state can resonate with a two electron state, because an electron and anti-electron are annihilating into a photon. Or any combination thereof. Therefore, all we really need are the field operators' equations of motion for the electron particle field and photon particle field.

Here is the dirac equation for the electronic field for electrons and their interactions with photons.

$$\hat{O}_e \Psi_e = \partial_\mu \left(\frac{\partial \mathscr{L}_{QED}}{\partial \left(\partial_\mu \Psi_e \right)} \right) - \frac{\partial \mathscr{L}_{QED}}{\partial \Psi_e}$$

```
{hat O}_e %PSI_e = partial_%mu left( {partial Laplace_QED}over{partial
left( partial_%mu %PSI_e right)} right) - {partial Laplace_QED}over{partial
%PSI_e}
```

And here is the maxwell's equations for the optic field for photons and their interaction with electrons.

$$\hat{O}_\gamma \Psi_\gamma = \partial_\mu \left(\frac{\partial \mathscr{L}_{QED}}{\partial \left(\partial_\mu \Psi_\gamma \right)} \right) - \frac{\partial \mathscr{L}_{QED}}{\partial \Psi_\gamma}$$

```
{hat O}_%gamma %PSI_%gamma = partial_%mu left( {partial Laplace_QED}
over{partial left( partial_%mu %PSI_%gamma right)} right) - {partial
Laplace_QED}over{partial %PSI_%gamma}
```

It is quite amazing that the equations of motion themselves already contain this particle changing behavior within them. In fact, the reason behind this is simple. Everything is classical fields described using relativistic wave equations like the klein gordon equation, dirac equation, and proca equations. These fields are spread out entities that interact in a digital way. When we couple fields together, the effect is to make the field nonlinear, and as such the wave-packets can split or merge, creating the illusion of particles. For QED, the electrons and photons it describes is only the tip of a vast iceberg.

When nuclear physicists cracked open the atomic nucleus, and found protons, neutrons, and the whole plethora of subatomic particles that we call the subatomic zoo, scientists found out that electrons and photons were only a small low energy, long wavelength subsection of the full theory. The standard model was put forth back in the 1970's, and since then, every experiment confirmed or said yes to this model, and said no to everything else. Simulations done in QCD or quantum chromodynamics for quarks inside a proton, found exact agreeement between the particle spectrum that contains baryons and mesons of the particle zoo, and the standard model. This is a huge triumph.

Now that we have a way to make every field wavepacket excitation obey an equation that describes its own black hole, not only allows one to introduce the black hole particle duality, but because of this duality, it allows one to unify the full Poincare gauge theory with the particle content of this new and confirmed standard model. Since

the duality gives us a given spacetime geometry for any input, QED, standard model, or otherwise, means that we do not need to quantize a geometric field like the Einstein-Cartan theory. Instead we can solve for quantum gravity, by sidestepping it completely.

Now that we have found a way to couple gravity back in from the black hole duality, means that the only input is the particle content. QED as we explained has the particle content for the visible daily life experience kind of particles, namely electrons and photons. But atoms contain nuclei that obey the full standard model, not just QED. So instead of presenting the duality here, we just present the full lagrangian for the standard model. Just know that the particle content is an input to the duality, not an output of the theory. So here is the next section that gives you what you need for the full theory of everything!

Section 5.7: The Field Lagrangian for the Standard Model

We have already met the electrons and the photons. Electrons interact with photons, because they have a property known as the electric charge. The electric charge of electrons is either positive one or negative one. The matter electrons are all negatively charged. The positively charged electrons are the opposite of matter, known as antimatter. Matter particles and antimatter particles can cancel eachother out, in a violent release of light energy. We say that photons, particles of light, are carriers of the electromagnetic force. After all, light waves are electromagnetic waves. Photons are themselves electrically neutral. The electric charge is not the only charge that exists in the cosmos. There are two others, the flavor charges, and the color charges.

There are four flavors of fermions, the up and down quarks, and the charged and neutral leptons. The electrons are an example of charged leptons. Every up quark has an electric charge of positive two thirds, and every down quark has an electric charge of negative one third. Like electrons, the antimatter quarks also have opposite electric charges. Just like how photons interact with electric charge, there are three other bosons that interact with the flavor charges. These bosons carry a force known as the weak nuclear force, and interconvert flavors into one another. Unlike photons, which do not self interact, the bosons which carry the weak nuclear force, certainly do self interact. We call these bosons, the W plus, the W minus, and the Z zero.

Then there are the color charges. Quarks are the only fermions with color charge. They come in three varieties of red quarks, green quarks, and blue quarks. The gluons interact with these three color charges. They carry a force, known as the fundamental strong nuclear force. The leptons no not experience the strong nuclear force, because they are color neutral, and as such, do not interact with gluons. Gluons, like quarks, have a color charge. In fact gluons come in eight varieties, with each kind of gluon having a unique color charge. This means that like the W and the Z bosons, gluons self interact. Gluons have a color charge of a given color, say red, and a given anticolor of a different type, say anti green. So a possible color charge for a gluon is red anti-green.

As usual, there is more to it than this. Each flavor of fermion, has three possible amounts of matter, or mass. The up type quarks, are the up quarks, charm quarks, and top quarks. The down type quarks are the down quarks, strange quarks, and bottom quarks. Electrons also have two heavier versions, the muon, and the tau. Why? The answer is the interaction with a particle known as the Higgs boson. It is very heavy, and has an unusual spin of zero. All of the other bosons, interact with charges, and have spin one. The higgs bosons don't carry forces, but they do give mass to the other particles that have it. There are exceptions though. The neutral leptons, we call neutrinos, don't have mass, unlike all the other fermions. The photons are massless, along with the gluons, which also are massless. However like the higgs boson itself, the W and Z bosons do have a huge mass. Therefore, the higgs boson interacts with itself.

With a roadmap of exactly which fields exist, one can use what we learned so far in this book, to build the theory of everything we talked about in this chapter. Luckily, particle physicists do indeed have a field theory known as the standard model. Some scientists like Frank Wilchek and Sean Carroll, have dubbed it the Core Theory. The equations themselves are found from the field lagrangian that I give below:

Here is the space-time interval for the flat minkowski background.

$$\eta^{\mu\nu}=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}$$

```
{ %eta^{%mu%nu} = left(matrix{1 # 0 # 0 # 0 ## 0 # -1 # 0 # 0 ## 0 # 0 # -1 # 0
## 0 # 0 # 0 # -1}right) }
```

And the good old imaginary unit.

$$i^2=-1$$

```
i^2 = -1
```

Here we get to the gauge bosons themselves, it is written in terms of the field strength tensors, akin to the riemann curvature tensor of the gravitational field equations, these three parts each have a different symmetry group. Remember how important symmetry was in our discussion of poincare gauge theory. The internal symmetry is now dependent on which of the 3 other forces we are talking about.

$$\mathscr{L}_{Gauge\ Bosons}=-\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu}-\frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu}-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

```
{ laplace_{Gauge Bosons} = -{frac{1}{4}}{G^a_{%mu%nu} G^{a%mu%nu}} -{frac{1}
{4}}{W^a_{%mu%nu} W^{a%mu%nu}} -{frac{1}{4}}{B_{%mu%nu} B^{%mu%nu}} }
```

For the first and most complicated of the forces, is the fundamental strong nuclear force. Every capital G is a gluon field. The a index runs from 1 to 8, and tells you which of the 8 gluons we are talking about. The mu's and nu's are the same spacetime indeces from the minkowski metric.

$$G^a_{\mu\nu}=\partial_\mu G^a_\nu-\partial_\nu G^a_\mu-g_s f^{abc}G^b_\mu G^c_\nu$$

```
{G^a_{%mu%nu} = partial_%mu G^a_%nu - partial_%nu G^a_%mu - g_s f^abc G^b_%mu
G^c_%nu}
```

Since the gluons self-interact, we need the strength of this interaction, and its given by this constant value: g sub s.

$$g_s = 1.221$$

```
g_s = 1.221
```

Later on, when we include the interactions with the quarks, each of the 8 gluons will interact with the 3 types of quarks through these matrices, known as Gell-Mann matrices. Here is the interaction with the first type of gluon.

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

```
{ %lambda_1 = left( matrix{0 # 1 # 0 ## 1 # 0 # 0 ## 0 # 0 # 0 } right) }
```

Similiarly with the second type of gluon.

$$\lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

```
{ %lambda_2 = left( matrix{0 # -i # 0 ## i # 0 # 0 ## 0 # 0 # 0 } right) }
```

And so on for the third type of gluon.

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

```
{ %lambda_3 = left( matrix{1 # 0 # 0 ## 0 # -1 # 0 ## 0 # 0 # 0 } right) }
```

The fourth type of gluon.

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

```
{ %lambda_4 = left( matrix{0 # 0 # 1 ## 0 # 0 # 0 ## 1 # 0 # 0 } right) }
```

The fifth type of gluon.

$$\lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

```
{ %lambda_5 = left( matrix{0 # 0 # -i ## 0 # 0 # 0 ## i # 0 # 0 } right) }
```

The sixth type of gluon.

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

```
{ %lambda_6 = left( matrix{0 # 0 # 0 ## 0 # 0 # 1 ## 0 # 1 # 0 } right) }
```

The seventh type of gluon.

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

```
{ %lambda_7 = left( matrix{0 # 0 # 0 ## 0 # 0 # -i ## 0 # i # 0 } right) }
```

And finally the eighth type of gluon.

$$\lambda_8=\frac{1}{\sqrt{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}$$

```
{ %lambda_8 = {frac{1}{sqrt{3}}}left( matrix{1 # 0 # 0 ## 0 # 1 # 0 ## 0 # 0 # -2 } right) }
```

We can arrange the eight, 3 by 3 matrices into a gluon interaction vector of vector length 8.

$$\lambda^a=(\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5,\lambda_6,\lambda_7,\lambda_8)$$

```
{ %lambda^a = (%lambda_1,%lambda_2,%lambda_3,%lambda_4,%lambda_5,%lambda_6,%lambda_7,%lambda_8) }
```

Their commutation relations give the following results, which are useful for the gluon self-interactions we just saw. The f indeces tell you which of the 8 gluons are in the interaction. The strength is given by their value. So for gluons of type 1, 2, and 3, the strength is 1 times the strong coupling constant.

$$f_{123}=1$$

```
f_123 = 1
```

Then we have the six other types of gluon self interactions of one-half the strong coupling constant.

$$f_{147}=f_{165}=f_{246}=f_{257}=f_{345}=f_{376}=\frac{1}{2}$$

```
f_147 = f_165 = f_246 = f_257 = f_345 = f_376 = frac{1}{2}
```

And lastly we have two more types of gluon self interactions with root three over two times the strong coupling constant. These relations are essential for the gluons to convey the fundamental strong nuclear force to the fields that experience it.

$$f_{458}=f_{678}=\frac{\sqrt{3}}{2}$$

```
f_458 = f_678 = { frac{sqrt{3}}{2} }
```

Next we have a mashup of the electromagnetic force and weak nuclear force. The field strength tensor here is like that of the strong nuclear force, but instead of being 3 by 3 matrices with 8 types of gluon, they are 2 by 2 matrices, with 3 types of W boson, some which are mixtures between Z bosons and photons. So instead of the a index running from 1 to 8, like earlier, here there are three W electro-weak bosons, and as such the a index runs from 1 to 3.

$$W^a_{\mu\nu}=\partial_\mu W^a_\nu-\partial_\nu W^a_\mu-g_2\epsilon^{abc}W^b_\mu W^c_\nu$$

```
{W^a_{%mu%nu} = partial_%mu W^a_%nu - partial_%nu W^a_%mu - g_2 %epsilon^abc W^b_%mu W^c_%nu}
```

And then we have a B electro-weak boson field, that is does not self interact. It also is a mixture of photon field and Z boson field.

$$B_{\mu\nu}=\partial_\mu B_\nu-\partial_\nu B_\mu$$

```
{B_{%mu%nu} = partial_%mu B_%nu - partial_%nu B_%mu}
```

Here is the definition of the first W boson field from the W plus and W minus bosons.

$$W^1_\mu = \frac{W^+ + W^-}{\sqrt{2}}$$

```
{ W^1_%mu = frac{W^{+{}} + W^{-{}}}{sqrt{2}} }
```

And a similar defintion for the second W boson field, also from the W plus and minus bosons.

$$W^2_\mu = \frac{W^- - W^+}{i\sqrt{2}}$$

```
{ W^2_%mu = frac{W^{-{}} - W^{+{}}}{i sqrt{2}} }
```

Then we have the on diagonal entries of the 2 by 2 matrices which leads us to the defintion of the third W boson field: a mixture of the Z boson and photon, the mixing angle given by the theta W parameter.

$$W^3_\mu = Z_\mu \cos\theta_W + A_\mu \sin\theta_W$$

```
{W^3_%mu = Z_%mu cos %theta_W + A_%mu sin %theta_W}
```

The same mixing also applies to the B field, also being a mixture of a photon and Z boson.

$$B_\mu = -Z_\mu \sin\theta_W + A_\mu \cos\theta_W$$

```
B_%mu = -Z_%mu sin %theta_W + A_%mu cos%theta_W
```

Here is the coupling constant between the electron-positron field, and the photon field. Sometimes called the fine structure constant, it sets the strength of the electromagnetic force.

$$e = \sqrt{\frac{4\pi}{137.036}}$$

```
{ e = sqrt{frac{4%pi}{137.036}} }
```

Here is the squared sine value for the weak mixing angle.

$$\sin^2\theta_W = 0.2315$$

```
{ sin^2 %theta_W = 0.2315 }
```

The B electro-weak field strength.

$$g_1 = \frac{e}{\cos\theta_W}$$

```
{g_1 = {frac{e}{cos %theta_W}} }
```

The W electro-weak field strength.

$$g_2 = \frac{e}{\sin\theta_W}$$

```
{g_2 = {frac{e}{sin %theta_W}} }
```

Here is the 2 by 2 matrix for the first W field. Also called Pauli-spin matrices, the relation would soon be clear, when we use these matrices for the dirac equation. But here it is quite amazing that the same 2 by 2 matrices are used for defining the three W electro-weak boson fields.

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

```
{%tau_1 = left(matrix{0 # 1 ## 1 # 0}right)  }
```

Here is the second Pauli matrix for the second W field.

$$\tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

```
{%tau_2 = left(matrix{0 # -i ## i # 0}right) }
```

And lastly the third Pauli matrix for the third W field.

$$\tau_3=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

```
{%tau_3 =  left(matrix{1 # 0 ## 0 # -1}right) }
```

In analogy with the 8 types of gluons, we have 3 types of electro-weak W bosons.

$$\tau^a=(\tau_1,\tau_2,\tau_3)$$

```
{%tau^a = ( %tau_1, %tau_2, %tau_3 )}
```

The commutation relations show that the three types of electro-weak W bosons self interact.

$$\epsilon_{123}=\epsilon_{231}=\epsilon_{312}=1$$

```
%epsilon_123 = %epsilon_231 = %epsilon_312 = 1
```

The same is true in reverse.

$$\epsilon_{321}=\epsilon_{132}=\epsilon_{213}=-1$$

```
%epsilon_321 = %epsilon_132 = %epsilon_213 = -1
```

Now we get to the Dirac kinetic term for all leptons and their interactions with the electro-weak bosons. Here is the lagrangian.

$$\mathscr{L}_{Lepton\,Kinetic}=(\bar{\nu}_L,\bar{e}_L)\tilde{\sigma}^\mu i D_\mu\begin{pmatrix}\nu_L\\ e_L\end{pmatrix}+\bar{e}_R\sigma^\mu i D_\mu e_R+\bar{\nu}_R\sigma^\mu i D_\mu \nu_R$$

```
{ laplace_{Lepton Kinetic} = ({bar %nu}_L,{bar e}_L) {tilde %sigma}^%mu  i
D_%mu left(matrix{ %nu_L ## e_L }right) + {bar e}_R %sigma^%mu i D_%mu e_R +
{bar %nu}_R %sigma^%mu i D_%mu %nu_R }
```

For reference we have the 2 by 2 identity matrix.

$$I_2=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

```
{I_2 =  left(matrix{1 # 0 ## 0 # 1}right) }
```

For right handed Dirac fermions, the Pauli-spin matrices are positive.

$$\sigma^\mu=(I_2,\tau^a)$$

```
{ %sigma^%mu = (I_2 , %tau^a) }
```

For left handed Dirac fermions, the Pauli-spin matrices are negative, following from the Minkowski metric. So here instead of a given 2 by 2 Pauli matrix referencing a given W field, we are referencing a particular coordinate axis in space, hence the mu.

$$\tilde{\sigma}^\mu=(I_2,-\tau^a)$$

```
{ {tilde %sigma}^%mu = (I_2 , - %tau^a) }
```

Again we have the 4 by 4 identity matrix for reference.

$$I_4=\begin{pmatrix}I_2 & 0\\ 0 & I_2\end{pmatrix}$$

```
{I_4 =  left(matrix{I_2 # 0 ## 0 # I_2}right) }
```

To show exactly how the commutation relations worked out for the gluons and W electro-weak bosons, as an example, we explain in analogy to the gamma matrices for the dirac equation, which you remember allowed us to solve the black hole – particle duality. This is why symmetry is important, as it simplifies complexity into simplicity. The gamma matrices here are not the same as before, but that is fine, given that we stick to our conventions, one way or another.

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \widetilde{\sigma}^\mu & 0 \end{pmatrix}$$

```
%gamma^%mu = left(matrix{ 0 # %sigma^%mu ## {tilde %sigma}^%mu # 0 }right)
```

The gamma matrices anticommute into the minkowski metric tensor, just as usual.

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2 I_4 \eta^{\mu\nu}$$

```
%gamma^%mu %gamma^%nu + %gamma^%nu %gamma^%mu = 2 I_4 %eta^{%mu%nu}
```

Here we have the covariant derivatives for the left handed electrons and left handed neutrinos. Notice how the W plus and W minus bosons convert these electrons and neutrinos into one another, and how the electric charge of the electron is coupled to photons with the fine structure constant, and not the neutrinos.

$$D_\mu \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = \left[\partial_\mu - \frac{i g_1}{2} B_\mu + \frac{i g_2}{2} W_\mu^a \tau^a \right] \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

```
{ D_%mu left(matrix{ %nu_L ## e_L }right) = left[{partial_%mu - {frac{i g_1}
{2}} B_%mu + {frac{i g_2}{2}} W^a_%mu %tau^a }right] left(matrix{ %nu_L ##
e_L }right) }
```

For right handed neutrinos the covariant derivative is just a partial derivative, showing its lack of interaction with the gauge bosons.

$$D_\mu \nu_R = \partial_\mu \nu_R$$

```
{ D_%mu %nu_R = partial_%mu %nu_R }
```

For the right handed electrons, there definitely is an interaction with the photon and Z bosons, given by the electro-weak B field coupling.

$$D_\mu e_R = \left[\partial_\mu - i g_1 B_\mu \right] e_R$$

```
{ D_%mu e_R = left[ partial_%mu - i g_1 B_%mu right] e_R  }
```

A similarity between quarks and leptons, can be seen here. There are two key differences. The first is that all quarks also have 3 components that interact with the 8 types of gluons, through the Gell-Mann matrices. The second is more subtle and is that the up and down types of quark fields, are again mixtures of each other.

$$\mathscr{L}_{Quark\ Kinetic} = (\bar{u}_L, \bar{d}_L) \widetilde{\sigma}^\mu i D_\mu \begin{pmatrix} u_L \\ d_L \end{pmatrix} + \bar{d}_R \sigma^\mu i D_\mu e_R + \bar{u}_R \sigma^\mu i D_\mu u_R$$

```
{ laplace_{Quark Kinetic} = ({bar u}_L,{bar d}_L) {tilde %sigma}^%mu  i D_%mu
left(matrix{ u_L ## d_L }right) + {bar d}_R %sigma^%mu i D_%mu e_R + {bar u}_R
%sigma^%mu i D_%mu u_R }
```

Here with the covariant derivative for left handed up and down type quarks, we see how this mixture manifests itself through experiments. When a top quark, the heaviest up type quark decays, it doesn't just decay into the heaviest down type quark. No! It decays into a mixture of all three masses of down quarks. We call the three mass types, generations which are field defintion mixtures like with the electroweak B fields being mixtures of the photon and Z bosons, and this is to not be confused with the three types of interaction with the gluons by the Gell-Mann matrices. This has nothing to do with mass and decay, and everything to do with the interaction between quarks and gluons. This property is called color charge.

$$D_\mu \begin{pmatrix} u_L \\ d_L \end{pmatrix} = \left[\partial_\mu + \frac{i g_1}{6} B_\mu + \frac{i g_2}{2} W_\mu^a \tau^a + \frac{i g_s}{2} G_\mu^a \lambda^a \right] \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

```
{ D_%mu left(matrix{ u_L ## d_L }right) = left[{partial_%mu + {frac{i g_1}{6}} B_%mu + {frac{i g_2}{2}} W^a_%mu %tau^a  + {frac{i g_s}{2}} G^a_%mu %lambda^a } right] left(matrix{ u_L ## d_L }right) }
```

The mixtures cancel out for right handed up type quarks, leaving us with interactions between quarks and gluons through the color charge interacting with the 8 types of gluons, through these Gell-Mann matrices.

$$D_\mu u_R = \left[\partial_\mu + \frac{2 i g_1}{3} B_\mu + \frac{i g_s}{2} G_\mu^a \lambda^a \right] u_R$$

```
{ D_%mu u_R = left[ partial_%mu + {frac{2 i g_1}{3}} B_%mu + {frac{i g_s}{2}} G^a_%mu %lambda^a right] u_R  }
```

The exact same holds for the right handed down type quarks. Notice how the up quarks have electric charge of plus two thirds, and how the down quarks have electric charge of minus one thirds.

$$D_\mu d_R = \left[\partial_\mu - \frac{i g_1}{3} B_\mu + \frac{i g_s}{2} G_\mu^a \lambda^a \right] d_R$$

```
{ D_%mu d_R = left[ partial_%mu - {frac{i g_1}{3}} B_%mu + {frac{i g_s}{2}} G^a_%mu %lambda^a right] d_R  }
```

Next up is the Higgs boson field lagrangian. The kinetic term gives the W minus, W plus, and Z zero bosons their mass, and the mass term gives the Higgs boson its own mass, by use of the quadratic potential, that makes the higgs field vibrate around a non-zero value.

$$\mathscr{L}_{Higgs\ Boson} = \overline{(D_\mu \phi)} D^\mu \phi - \frac{m_h^2}{2 v^2} \left[\bar{\phi} \phi - \frac{v^2}{2} \right]^2$$

```
{ laplace_{Higgs Boson} = { { overline {(D_%mu %phi)} } D^%mu %phi - {frac{m^2_h} {2 v^2}} left[{{bar %phi} %phi - {frac{v^2}{2}} }right]^2 } }
```

Here we define this non-zero value as another field refinition. So we refined the fields for the electro-weak bosons, the quark field generations, and now for the higgs boson field itself.

$$\phi = \begin{pmatrix} \frac{v+h}{\sqrt{2}} \\ 0 \end{pmatrix}$$

```
{ %phi = left({matrix{{frac{v+h}{sqrt{2}}} ## 0}}right) }
```

The interaction of the higgs boson with the gauge bosons for the weak nuclear force and given by this covariant derivative of the full higgs field in its kinetic term.

$$D_\mu \phi = \left[\partial_\mu + \frac{i g_1}{2} B_\mu + \frac{i g_2}{2} W_\mu^a \tau^a \right] \phi$$

```
{ D_%mu %phi = left[{partial_%mu + {frac{i g_1}{2}} B_%mu + {frac{i g_2}{2}} W^a_%mu %tau^a }right] %phi }
```

For this we have the higgs boson mass itself in MeV.

$$m_h = 125090$$

```
m_h = 125090
```

And its average settled field value again in MeV.

$$v = 246000$$

```
v = 246000
```

Last, but not least, is the Yukawa interaction between the higgs boson and the left and right handed fermions. This is the only interaction that couples the left and right handed fermionic fields together, giving all quarks and charged leptons their masses.

$$\mathscr{L}_{Fermionic\ Mass}=\left(1+\frac{h}{v}\right)\left[\bar{u}\,M_u u+\bar{d}\,M_d d+\bar{e}\,M_e e\right]$$

```
{ laplace_{Fermionic Mass} = left(1 + {frac{h}{v}} right) left[{{bar u} M_u u + {bar d} M_d d + {bar e} M_e e }right] }
```

The electron field a vector of length 2 of both the left and right handed parts.

$$e=(e_R,e_L)$$

```
e = ( e_R , e_L )
```

The same goes for the neutrino fields.

$$\nu=(\nu_R,\nu_L)$$

```
%nu = ( %nu_R , %nu_L )
```

However, for the three generations of down type quarks, known as down, strange, and bottom quarks, they are mixtures between all three generations. But they are there nonetheless. This is why when the left handed quarks decayed they can cross generations by exchanging W plus and W minus bosons. But in this mass lagrangian, the generation mixing cancels itself out, leaving us with the left and right handed down quarks, with mass, similar to the electrons gaining mass. But the generation mixing is still there hidden inside the field redefinitions.

$$d=(\boldsymbol{U}_R^d d_R,\boldsymbol{U}_L^d d_L)$$

```
d = ( {bold U}^d_R d_R , {bold U}^d_L d_L )
```

The same trickery is used for the up quarks. Their three generations are the up, charm, and top quarks. As before this cancels out, giving the usual linear mass relations between each mass type as a pure left handed to right handed interaction.

$$u=(\boldsymbol{U}_R^d u_R,\boldsymbol{U}_L^d u_L)$$

```
u = ( {bold U}^d_R u_R , {bold U}^d_L u_L )
```

When we expand the lagrangian in full, interactions between W bosons show up from earlier, so having a matrix that mixes generations between left handed up quarks and left handed down quarks is useful. It is defined as the product of those two bold U matrices.

$$V^q=\boldsymbol{U}_L^u\boldsymbol{U}_L^d$$

```
V^q = {bold U}^u_L {bold U}^d_L
```

Here is another way to write it. As a matrix product of five 3 by 3 matrices. It gives the strength of interaction between the W plus boson and the 3 generations of up type quarks coverting into the 3 generations of the down type quarks. The s and c are sine and cosine of some new angles. Not just the weak mixing angles. The delta is a phase angle.

$$V^q=\begin{pmatrix}1&0&0\\0&c_{23}&s_{23}\\0&-s_{23}&c_{23}\end{pmatrix}\begin{pmatrix}e^{-i\delta/2}&0&0\\0&1&0\\0&0&e^{i\delta/2}\end{pmatrix}\begin{pmatrix}c_{13}&0&s_{13}\\0&1&0\\-s_{13}&0&c_{13}\end{pmatrix}\begin{pmatrix}e^{i\delta/2}&0&0\\0&1&0\\0&0&e^{-i\delta/2}\end{pmatrix}\begin{pmatrix}c_{12}&s_{12}&-s_{12}\\c_{12}&0&0\\0&0&0\end{pmatrix}$$

```
V^q = left( matrix{1 # 0 # 0 ## 0 # c_23 # s_23 ## 0 # -s_23 # c_23 } right)
left( matrix{e^{-i%delta/2} # 0 # 0 ## 0 # 1 # 0 ## 0 # 0 # e^{i%delta/2} }
right)left( matrix{c_13 # 0 # s_13 ## 0 # 1 # 0 ## -s_13 # 0 # c_13 }
right)left( matrix{ e^{i%delta/2} # 0 # 0 ## 0 # 1 # 0 ## 0 # 0 #
e^{-i%delta/2} } right)left( matrix{c_12 # s_12 # -s_12 ## c_12 # 0 # 0 ## 0 #
0 # 0 } right)
```

Here is the delta phase angle is given in radians.

$\delta = 1.2042$

```
%delta = 1.2042
```

Here is the conversion between sine and cosine for the other angles.

$c_j = \sqrt{1 - s_j^2}$

```
c_j = sqrt{1-s_j^2}
```

Then we get to the sine of the angles themselves. This angle is quite large, showing that charm and strange quarks, can decay into up and down quarks with a lot of mixing. These are the first and second generations respectively.

$s_{12} = 0.2253$

```
s_12 = 0.2253
```

This is the sine of the angle mixing the second and third generations, and its much smaller.

$s_{23} = 0.041$

```
s_23 = 0.041
```

And this is the sine of the angle mixing the first and third generations, and is smaller still.

$s_{13} = 0.0035$

```
s_13 = 0.0035
```

Here are the masses for the unmixed charged lepton fields. Neutrinos on the other hand do not interact with the higgs boson, and as such are massless.

$$M_e = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

```
{M_e = left( matrix{ m_e # 0 # 0 ##  0 # m_%mu # 0 ## 0 # 0 # m_%tau } right)}
```

Here is the unmixed mass matrix for the up type quarks.

$$M_u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}$$

```
{M_u = left( matrix{ m_u # 0 # 0 ##  0 # m_c # 0 ## 0 # 0 # m_t } right)}
```

And here is the unmixed mass matrix for the down type quarks.

$$M_d = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}$$

```
{M_d = left( matrix{ m_d # 0 # 0 ##  0 # m_s # 0 ## 0 # 0 # m_b } right)}
```

In MeV is the mass of the electron.

$m_e = 0.51099895069$

```
m_e = 0.51099895069
```

Again in MeV, is the mass of the muon.

$$m_{\mu}=105.658367$$

```
m_%mu = 105.658367
```

The mass of the tau lepton.

$$m_{\tau}=1776.84$$

```
m_%tau = 1776.84
```

Up quark mass.

$$m_{u}=2.01$$

```
m_u = 2.01
```

Down quark mass.

$$m_{d}=4.79$$

```
m_d = 4.79
```

Strange quark mass.

$$m_{s}=93.5$$

```
m_s = 93.5
```

Charm quark mass.

$$m_{c}=1273$$

```
m_c = 1273
```

Bottom quark mass.

$$m_{b}=4183$$

```
m_b = 4183
```

Finally we have the top quark mass.

$$m_{t}=172560$$

```
m_t = 172560
```

The core theory lagrangian is the sum of many parts. We mentioned them earlier in this chapter. Gauge bosons include the photon, the W plus and minus bosons, the Z zero boson, and the 8 types of gluons. The gauge boson fields and their interactions are given by the first term in the lagrangian. The fermions include both quarks and leptons. Unlike leptons, quarks have electric charges of thirds and interact with gluons. Both quarks and leptons are fermions, and only their left handed counterparts interact with the W and Z bosons. These interactions that the gauge bosons have with the fermionic fields are split between leptons and quarks. The lagrangians for both quarks and leptons also have kinetic terms, which include the dirac equations without their mass terms. The higgs boson also has a lagrangian, that makes all massive fields to vibrate and hence gives them their mass. The higgs boson term also gives mass to all bosons that have it. The last term is the fermionic mass term, that allows the higgs boson to give mass to the quarks and leptons themselves. But together it's a beast. But here we have it, the full core theory field lagrangian.

$$\mathscr{L}_{core}=\mathscr{L}_{Gauge\ Bosons}+\mathscr{L}_{Lepton\ Kinetic}+\mathscr{L}_{Quark\ Kinetic}+\mathscr{L}_{Higgs\ Boson}+\mathscr{L}_{Fermionic\ Mass}$$

```
laplace_core = laplace_{ Gauge Bosons } + laplace_{ Lepton Kinetic } +
laplace_{ Quark Kinetic } + laplace_{ Higgs Boson } + laplace_{ Fermionic
Mass }
```

First it was at CERN, where team leaders announced the Higgs boson discovery. Then, came the confirmation of the standard model lagrangian. So far so good. So with

this in mind, here is the story of how everything emerges from the very theory I completed. Here we go!

Chapter 6: A Lennard Jones Fluid

The scale of the universe is far beyond our day to day experience. To find out what happens in such domains, we need to grasp the three pillars of modern science: gravity, light speed, and quantum waves. Black holes render compatible these three pillars into a single mathematical framework. Because of this unity, fundamental particles with mass and spin are all rotating black holes. This very definition follows straight from the idea that the geometry of space and time is all there is to know about reality. In fact, all of these fundamental particles are nothing more than discrete ripples in fields. The quantum physics of particles, and its observer effect, all emerge from a universe made of fields. In a sense, everything including light, electricity, and matter comes from the shape of nothing itself. A scientist once said: "I think we have it." We finally have the theory of everything!

From this theory, one can compute everything from the way viscous fluids flow to the chemical reaction rates of the elements made in the big bang. Inter-molecular potentials determine how the behavior of atoms in motion within solids, liquids, and gasses, gives rise to material phenomena such as movement, sound, and heat. As it turns out, it is interesting to look at how gravity affects this fluid motion on a large scale, forming the cosmic web, galaxies, stars, and worlds with life. If one took a census of everything in the universe and weighed it, one would find out that 84% is primordial black holes, 12% is protons of Hydrogen-1, and 4% is alpha particles of Helium-4. The atomic structures of hydrogen and helium, are formed from their interactions with both electrons of electricity and photons of light. All of this amazing complexity comes about from processes that happened in the first moments after the big bang. And yet, from all of us, all of this might just be a computer simulation…

Section 6.1: Atoms in Motion

All material things are made of atoms. All atoms move in perpetual motion. They weakly attract one another when close, but repel each other strongly when squeezed too close. This pattern is known as an inter-molecular potential, and is key to understanding the phenomena of solids, liquids, and gasses.

In gasses, the atoms are scattering off one another in all directions, colliding like a game of cosmic pinball. This is what a gas actually is. But, on the other extreme we have solids. In solids, the atoms are all stuck together in a regular lattice of rows and columns, and cannot move around. In liquids however, the atoms are able to move around, sliding past each other, but cannot break free all together. This is what makes something a liquid.

With an intuitive grasp of atoms in motion, it is interesting to look at the characteristics that this motion can take. For example, imagine a cluster of atoms vibrating at a bunch of different frequencies. Such waves in the atomic structure of a

material are called sound waves. Sound waves with a high frequency or pitch have a short wavelength, while sound waves with a low frequency and low pitch have long wavelengths.

Heat is also atoms in motion. In fact, it's the random motion on the scale of the atoms themselves. Therefore, the very highest frequencies of sound are actually modes of vibration that constitute the temperature of an object. In other words, remove all high pitched sounds within the material, including the sound waves with wavelengths on the atomic scale, and the temperature would drop to absolute zero.

The sound spectrum also continues to the low end with very long wavelengths and very low frequencies. We can also think of vibrating atoms in terms of their period of oscillation. A material object that moves back and forth slowly enough, has wavelengths so long, that they cover the entire object at a macroscopic scale. Since the atoms are moving in unison, its no longer a sound wave, but simply the motion of the object as a whole. Yes, even the atoms in a speeding bullet are moving as it flies.

So, as we can see, motion, sound, and heat, are all just different manifestations of atomic vibrations. Here is an equation that gives the potential energy of any pair of atoms or molecules. It is known as the Lennard Jones potential.

$$V_{LJ}=\frac{1}{r^{12}}-\frac{1}{r^{6}}$$

```
V_LJ = { {frac{1}{r^12}} - {frac{1}{r^6}} }
```

As we can see this equation explains our intuition perfectly. It has a hard inner wall, with a small dip right along its edge. When two atoms come near, they weakly attract, when they come too close, they strongly repel. But with a sufficient distance, they just coast along in constant speed straight lines.

Whoa. Where does that sound familiar. Oh, right… It is our CGH equation from earlier. Atoms are stable in the condition that their average kinetic energy or temperature doesn't get too high. So like how QED is a low energy version of the theory of everything. The lennard jones potential we have here, is the next step on that ladder of course graining to ever more effective theories. So as long as you don't zoom in too far inside an atom, it works well. Just like how QED works beyond the nucleus of any atom, this means that the Lennard Jones potential works beyond any atom's internal structure.

Section 6.2: How Gravity affects Large Scale Structure

However, one might imagine what happens if we include just a feeble touch of self-gravity to our inter-molecular potential. Given by Newton's laws of motion and universal gravitation, the interplay between gravity on our fluid will have some interesting effects.

On a small scale, gravity is negligible. But as we zoom out, the amount of atoms in our view increases, and the amount of gravity accumulates. Eventually, as we zoom out past the scale of entire worlds, and we can treat each world as a single object,

accelerating objects toward it, according to the inverse-square law in 3D space. It's this self gravity of large enough worlds, that makes them spherical, such as the Earth itself. And it is also the reason the solar system holds together. But don't even get me started on the complicated dynamics that gave birth to our solar system.

Take an expanding cloud of gas, with atoms that obey the inter-molecular potential and also Newton's gravity. The gasses first cool down, and tiny grains form, like a cloud of snow particles. As the cloud fragments, the particles collide and stick, with larger fragments orbiting around one another. Within a relatively short time, you end up with a cosmic web, filled with galaxies that look like spinning pinwheels, and within each galaxy, solar systems containing stars, planets, moons, asteroids and comets, all in orbit around each other. Not bad, because this is exactly what the large scale structure of our actual universe looks like in the first place.

Here is what such a touch of self gravity looks like in the lennard jones potential. Highly exxaggerated here is an example equation.

$$V_{LJ,G} = \frac{1}{r^{12}} - \frac{1}{r^{6}} - \frac{0.01}{r}$$

```
V_{LJ,G} = { {frac{1}{r^12}} - {frac{1}{r^6}} } - frac{0.01}{r}
```

Gravity just adds on. Like our black hole particle duality, which says that every fundamental particle can be treated as a black hole source. We already saw the utility in this for effective theories. Given that QED worked even if one includes the duality, the same applies for atoms, dust grains, bullets, and entire worlds like the Earth. Thats why we saw cosmic structure emerge, because that is what happens when we take simple laws of physics and see what emerges from them when we apply those same laws to larger and larger scales of physics.

Next, we have the following chapter about what we left out in this course graining procedure of ours. We ommitted atomic nuclei completely, in our discussion on QED. In fact, by including their effects on the electronic and optic fields we do see all around us, we can not only derive the strength and distance of the lennard jones interaction, but also all of chemistry. So lets go ahead and do that.

Chapter 7: Hydrogen and Helium

From this idea, all we need is an empty box, that we fill with a gaseous mix of atoms. But one question arises… What atoms do we use, and by how much? By studying the cosmic abundances of the elements, we can determine that by mass, 75% of the universe is Hydrogen-1, and 25% is Helium-4. Almost. However, there are tiny impurities that makes life possible. Lets take a closer look.

These impurities also are atoms. Bigger atoms. Atoms like Oxygen, Carbon, and Nitrogen, which are essential for the building blocks of life. They and all other atoms only adds up to another 2% of the universe by mass. If the total mass of all atoms is a metric ton, roughly only 20 kilograms of these impurities would be needed.

Lets begin with Oxygen. Oxygen is the 8th element on the periodic table. It is a very reactive and corrosive gas, as it needs 2 more electrons to complete its outer valence shell. Burning Hydrogen and Oxygen together, forms water vapor in an explosive crack. Oxygen also forms diatomic molecules, which constitute 21% of our atmosphere. So, air, water, and even rocks contain lots of oxygen. So lets add 10,420 grams of Oxygen-16 to our box.

Next up is Carbon, Carbon is the 6th element on the periodic table. Commonly seen in soot or charcoal as graphite, it needs 4 electrons to complete its outer shell. Hydrocarbons such as Methane, are what happens when carbon and hydrogen combine. Carbon Dioxide, are what happens when carbon and oxygen combine. In fact carbon can form more molecules and compounds, than all other molecules without carbon combined. Carbon is therefore the next element that is essential for life. Lets now add 4,680 grams of Carbon-12 to our box.

After this, elements get harder to find. In the case of Neon, it is a noble gas, in the sense that neon atoms don't bond with any other atoms, including themselves. Because its completely non-reactive, means that an atom of neon is a loner. It is the 10th element on the periodic table. Lets include 1,340 grams of Neon-20 to our box.

Iron is the 26th element of the periodic table. It is a transition metal. It is a very dense metal, usually likes to lose 2 or 3 electrons depending on the situation, and is found in abundance in the earths outer and inner core. On the surface, it is used as the main component of steel, and therefore modern bridges, ships and skyscrapers will be impossible without it. Lets add 1,090 grams of Iron-56 to our box.

Then there is Nitrogen, Nitrogen is the 7th element on the periodic table. The most natural form is of a diatomic gas, comprising 78% of our atmosphere, with Nitrogen atoms needing 3 electrons to complete their outer shells. In Ammonia, it is achieved by combining with Hydrogen. Nitrates are Nitrogen with Oxygen, and Cyanide is Nitrogen with Carbon. Not exactly pleasant, but is also a key component of amino acids in proteins, some vitamins, and even DNA. Lets thus add our 960 grams of Nitrogen-14 to our box.

Silicon is the 14th element of the periodic table. It is a semimetal, which means its properties are between that of a metal and a non-metal. It is this property that makes modern computers possible. Silicon tends to lose 4 electrons to Oxygen, forming a mineral known as Quartz. Glass crystal is just Quartz, and most of the Earth rocks in the Continental Crust are mainly Quartz. Lets add 650 grams of Silicon-28 to our box.

Magnesium is the 12th element of the periodic table. It is an alkaline earth metal. From its name, Magnesium tends to lose 2 electrons to Oxygen, and when combining with quartz minerals, forms a mineral known as Olivine, a green jewel that is a key component of most of the Earth's mass in its Mantle, even including some Oceanic crust. Lets add 580 grams of Magnesium-24 to our box.

Second to last is the stinkiest element, Sulfur. Sulfur is the 16th element of the periodic table. Some common forms of sulfur include hydrogen-sulfide, the gas with the characteristic rotten egg smell, and sulfuric acid which is found in car batteries, and is extremely corrosive. Sulfur combines with other elements in our bodies in many proteins, and a few vitamins. Lets add 440 grams of Sulfur-32 to our box.

Lastly, we have Argon. Like Neon, and Helium before it, it is a noble gas, comes as single atoms, and is completely inert and non reactive. It is the 18th element of the periodic table. Lets finish by adding 200 grams of Argon-36 to our box.

So in short, many of these elements make like possible. With a universe as large as ours, it would be quite strange if earth life is the only life out there. With Oxygen, Carbon, and Nitrogen, in the mix from the get go, alien life should be inevitable. But this is the last 2% of all atoms by mass. Hydrogen and Helium make the atomic majority.

Hydrogen-1 makes 75% of our metric ton box, and Helium-4 makes 25% of our metric ton box. Hydrogen-2 and Helium-3, although stable, are quite rare in terms of mass, but by number, 26.4 ppm is Hydrogen-2 and 10.3 ppm is Helium-3. Also by number this means that we have 750,000 ppm of Hydrogen-1, and 62,500 ppm of Helium-4. Hydrogen tends to form diatomic molecules, and Helium being a noble gas is an inert gas of single atoms. Hydrogen is the 1st element of the periodic table, and Helium is the 2nd element of the periodic table.

But! That is not all there is, and far from it. In fact, remember that all material things are atoms, but that isn't to say that other forms of energy exist out there that are not atoms. You might be surprised to know that 5 metric tons of primordial black holes have to be added to the box, in order to match cosmological observations. These black holes are essentially invisible and intangible, just like a ghost. This is because the invisible matter that holds galaxies together into the cosmic web, interacts only through gravity. As we know already, a rotating black hole is a particle with mass and spin, just like an electron in an atom or photon of light. In other words, the fleeting existence of quarks and gluons inside the atomic nuclei we discussed are also microscopic black holes. Everything is black holes! It's just that the black holes in atomic matter have properties

with such small mass, that they act more like ripples in fields, but nonetheless, the invisible matter that holds the galaxies of the cosmic web together is real.

The dark matter spectrum, is not known yet, as one would have to run detailed simulations of black holes. (again provided by the python test suite and a python black hole orbiter). The spectrum of masses for dark matter, is unknown as of today, but is likely a very broad spectrum, given that monochromatic masses have been ruled out in many domains. The black holes are definitely primordial, but their origin could be geometric if known. It is possible the spectrum is a uniform distribution at the big bang singularity or could be the result of quantum fluctuations in the later primordial plasma.

Now in order to know the formation and evolution of chemistry, we need to start at the basics. What are atomic nuclei made of? Lets tackle this next.

Section 7.1: The Abundance of Chemical Elements in the Universe

Here is a table showing the abundances of the most common nuclei in the universe at large, and what such a nucleus is made of.

Nucleus:	Constituents:
150,418 H-1	1 p
6 H-2	1 p + 1 n
2 He-3	2 p + 1 n
12,500 He-4	2 p + 2 n
77 C-12	6 p + 6 n
14 N-14	7 p + 7 n
133 O-16	8 p + 8 n
13 Ne-20	10 p + 10 n
1 Ne-22	10 p + 12 n
5 Mg-24	12 p + 12 n
5 Si-28	14 p + 14 n
2 S-32	16 p + 16 n
1 Ar-36	18 p + 18 n
4 Fe-56	26 p + 30 n

1 proton is Hydrogen-1, 2 protons and 2 neutrons is Helium-4. And so on, all the way down to the 4.9 ppm level. You can compute the total abundances and you will find the numbers we talked about in the introduction to this chapter on Hydrogen and Helium. Next, lets define the chemistry of this nuclear sample.

Section 7.2: A Cosmic Chemical Formula

Here is a table showing the exact proportions of every atom, molecule, and mineral, to the same 4.9 ppm level. I know I put the nuclear mass at 1 gram. Its an

atomic mass unit, but its useful for chemists who are used to the molar amounts. Anyhow, with that issue aside, one has a full inventory of the universe. These compounds are ubiquitous in deep space, inside the earth, and in comets, including the nebula that formed our solar system. Here it is.

Chemical Formula:	Amount:	Name:
75,000 (H-1)2	150 kg	Hydrogen Gas
12,500 He-4	50 kg	Helium Gas
80 (H-1)2(O-16)	1,440 g	Water Vapor/Ice
55 (C-12)(H-1)4	880 g	Methane Gas/Ice
19 (C-12)(O-16)2	836 g	Carbon Dioxide Gas/Ice
3 [(Mg-24)(Si-28)(O-16)3]	300 g	Enstatite Rock
13 Ne-20	260 g	Neon Gas/Ice
10 (N-14)(H-1)3	170 g	Ammonia Gas/Ice
3 [(Fe-56)]	168 g	Iron Metal
1 [(Mg-24)2(Si-28)(O-16)4]	140 g	Forsterite Rock
1 [(Fe-56)(S-32)]	88 g	Troilite Ore
1 [(Si-28)(O-16)2]	60 g	Quartz Rock
2 (N-14)2	56 g	Nitrogen Gas/Ice
1 Ar-36	36 g	Argon Gas/Ice Isotope
3 [(C-12)]	36 g	Graphite
1 (H-1)2(S-32)	34 g	Hydrogen Sulfide Gas/Ice
1 Ne-22	22 g	Neon Gas/Ice Isotope
6 (H-2)(H-1)	18 g	Hydrogen Gas Isotope
2 He-3	6 g	Helium Gas Isotope

Now that we have an idea of what things form from our cosmic soup of nuclei. It is time to turn out attention inwards again, back to the theory of everything that makes it all work.

Section 7.3: Atomic Structures and Substructures

As it turns out, every atom of the periodic table consists of a heavy and dense, positively charged nucleus, surrounded by two fields, the electronic field and the optic field. The discrete vibrations of the electronic field are the negatively charged lowmass electrons in atoms, while the discrete vibrations of the optic field are the photons of light, which carry the electric and magnetic forces from one place and time to another. Put together, electrons and photons create all of the solid state and chemical forces within any substance.

Within an atom, the electrons fill "shells" or orbitals two electrons at a time. Each of these orbitals is a standing wave of the electronic field. Since the nucleus is heavy and positively charged, it creates a sink for electrons to be attracted to. The reason why electrons fill one orbital at a time is because they are anti-symmetric under

exchange. This is what makes an electron a particle of matter. For comparison, photons of light carry the electric and magnetic forces, because they are symmetric under exchange. Ultimately the reason why electrons are anti-symmetric, rather than symmetric, is tied to the fact that the electronic field itself only transforms half as much as the optic field does under a change in spacetime perspective. But enough about electrons and photons, what is the atomic nucleus anyways? After all, if electrons and photons were all there is, there would be only a single type of atom: Positronium and nothing else, and even then, given that Positronium atoms annihilate by themselves in less than a billionth of a second, what gives?

The answer lies in the fact that there are roughly 250 stable nuclei, and thousands more unstable nuclei. Once we zoom inside the nucleus, we understand that it is a chaos of nucleons and pions. It is said that nucleons are quark triplets, and pions are quark-antiquark pairs. But, in reality, the nucleus is a swarm of quarks and gluons, that are discrete waves in their respective fields, that only resonate fully as composite units. There is much to know as of yet about the field equations, such as their non linear behaviors, which are important to the structure and function of various nuclei, such as properties like radioactivity and binding energies. But all of the nuclei would not exist, without a chain of events of the early universe, known as the big bang.

Common Stable Particles:

For reference, here are the most common stable particles you will find in the universe at large. The table shows their relative abundance, and their properties like mass and spin. Since we are talking about nuclei with QED intact, we include the electric charge as well for completeness. Just know that this charge(like any other) doesn't participate in the black hole duality. Here is the table:

name:	# Abundance:	Mass in MeV:	Charge:	Spin:
alpha particles	1	3727.3794118	+2 e	0
protons	12	938.27208943	+1 e	½ hbar
electrons	14	0.51099895069	-1 e	½ hbar
neutrinos	18 * 10^9	0	0	½ hbar
photons	22 * 10^9	0	0	1 hbar

Now we can see the exact properties of the common bulk matter and radiation that makes up our universe at large. Next up are the unstable particles.

Unstable Particles:

This table is the same as the last, but instead of an abundance, we have an average lifetime. These particles are either unstable constituents or unstable byproducts of those constituents or both. Here is the table.

name:	Lifetime:	Mass:	Charge:	Spin:
neutrons	880.2 s	939.56542194 MeV	0	½ hbar
muons	2.197 us	105.6583755 MeV	-1 e	½ hbar
pions	26 ns	139.57039 MeV	+1 e	0

Then give that these particle are unstable, knowing what they decay into is of need. So here are the decay chains of these unstable particles.

Decay Chain:

neutron → proton + electron + electron-anti-neutrino

muons → electron + electron-anti-neutrino + muon-neutrino

pion → anti-muon + muon-neutrino

We can see that we already have defined the properties of the decay products too, so if you need to, like before, look back to any previous table. Next, are the consitutent particles that make up another composite particle.

Constituents:

Then for all composite particles, we note what it is made up of, its constituents, and not just what came out, but how much energy was required to break open the composite to reveal such a given constituent. And not only that, but the force field that mediates such a binding interaction. Here is the table for that.

composite = its constituent parts:	Mediator:	Energy Debt:
hydrogen molecule = 2 hydrogen atoms	2 photon fields	-4.52 eV
helium atom = 1 helium ion + 1 electron	2 photon fields	-24.6 eV
helium ion = 1 alpha particle + 1 electron	2 photon fields	-54.4 eV
hydrogen atom = 1 proton + 1 electron	2 photon fields	-13.6 eV
alpha particle = 2 protons + 2 neutrons	3 pion fields	-28.3 MeV
proton = 2 up quarks + 1 down quark	16 gluon fields	confined
neutron = 2 down quarks + 1 up quark	16 gluon fields	confined
pion = 1 up quark + 1 down anti-quark	16 gluon fields	confined

Now that we have an idea of what can form chemically when nuclei are included, to what these nuclei are made of and what they do, it is time to play an interesting thought game. What happens when we apply our theory of everything to reality itself. What will happen, given what we know? Lets find out…

Section 7.4: The Big Bang Theory of Cosmic Evolution

The big bang theory is not just a hypothesis like my black hole idea, and it isn't a TV show either. This is the story of how everything came to be. What would genesis look like if it were run by my theory. As we shall see, extremely well. Lets now run through the gauntlet, and for the first time ever, find out how everything emerges out of nothing. Here we go!!!

The creation of everything

Before the beginning, there was absolutely nothing: No space, no time, no matter. And from nothing emerges everything. Thus, in the beginning, everything is an infinitely dense, infinitely hot, singular point of origins. This mysterious point, is what created everything that follows it, and obeys a set of rules known as the laws of physics.

Lets now pause for a moment and imagine. What kinds of things will emerge from the physics? How will fluids and chemistry come about from mathematics alone? In astronomy, how does order come from the chaos of waves? But for now, to be able to grasp everything that follows, we need to know a few things about physics.

The theory of everything is also known as the unified field theory, because that is what reality is made of: Waves! Put together, these waves described by the unified physics of Special Relativity, Universal Gravitation, and Quantum Mechanics, determine how the entire universe will be built. Now, to see its effect in action, lets turn our attention to the following sequence of events…

In the first tiniest fraction of an instant, the universe explodes into existence, as a rapidly expanding, tremendously hot and dense firestorm, spread uniformly across space, and that extends infinitely in all directions. It is like dropping a nuke in every cubic micron of space, but then to realize that your cubic micron is now a cubic kilometer, as space itself, ever more slowly, continues to expand out in all directions. The universe continues to expand and cool.

In just another few tiny fractions of an instant or so, the tremendously hot and dense, but uniformly spread firestorm that is infinite in extent, has expanded, cooled, and diluted, into a not so hot, not so dense firestorm, that continues to be uniformly spread, and infinite in extent. For some actual numbers, given that our "not so hot" universe, now means ten quadrillion degrees, imagine how incomprehensibly hot the universe was right at the start: ten quadrillion times hotter than ten quadrillion degrees! The universe continues to expand and cool.

So. With this in mind, lets now track the evolution of our particles. Every particle in existence, is constantly smashing into other particles. We say that the particles are in thermal equilibrium. The average kinetic energy per particle, is uniform throughout the cosmos. Looking at the sound waves within the aether, we can see the waves are vibrating with an extremely high frequency. We will measure frequency with a new system of units. The electron volt. Its written as, "eV". Electron volts are useful because they can be understood as, energies, momenta, frequencies, and wavenumbers. In quantum field theory, everything is measured in electron volts. Light speed is 1 foot per nanosecond, and one electron volt of energy corresponds to a wavelength of 1.24 microns. As of now, the weak interaction and the electromagnetic interaction, are one and the same. Moreover, every particle is massless, and moves at the speed of light. This will change soon, when the temperature drops below 246 GeV. The universe continues to expand and cool.

<u>The higgs boson activation</u>

Passing 1 femtosecond, all of the quarks, antiquarks, leptons, antileptons, and the bosons, are almost in thermal equilibrium, and haven't yet aquired mass from this so called higgs boson. Yet after 10 picoseconds, the temperature drops below the threshold

of 246 GeV, and the higgs boson activates. The electromagnetic force carried by photons, is now distinct from the weak nuclear force, carried by the W plus, W minus, and Z bosons. These intermediate vector bosons, along with the higgs boson itself become very massive, on the order of 80 GeV, 92 GeV, and 125 GeV respectively. Since the electroweak bosons interact between themselves, we would expect to find confined states of these bosons. However, now that some of these electroweak bosons have a great deal of mass, removes any possibility of confinement. However these are not the only massive particles around. Every flavor of quark now has three possible masses. There are the up, charmed, and top flavors, and then the down, strange, and bottom flavors. The same applies to the charged leptons, which are the electron, the muon, and the tau. The neutral leptons, the neutrinos, do not have mass, and as such continue to move at light speed. The three top quarks, which are the heaviest flavor of the up type quarks, have masses of 173 GeV. That is much heavier than the W, Z, and the higgs boson. The universe continues to expand and cool.

After the next few hundred picoseconds, the top quarks and antiquarks, higgs bosons, Z bosons, and the W plus and minus bosons, are all gone. The energy of their annhililation, gets dumped into the remaining lighter particles. With these particles out of the picture, the weak interaction, reemerges as a short ranged force, 100 times shorter than a femtometer. In contrast, the quarks and gluons, are starting to feel their presence. This is why we call this the strong nuclear force. This force has litterally several metric tons of strength. The universe continues to expand and cool.

As we pass the ten nanosecond mark, the 4.5 GeV bottom quarks, 1.7 GeV tau leptons, and the 1.2 GeV charm quarks, annhililate away next. This seems like a losing battle, as all of the precious universe building particles, are just dissapearing, one by one, until nothing is left. After all of this, the only massive particles left are, the strange quarks, muons, down quarks, up quarks, and electrons, along with their antiparticles. So, to summarize. In these first instants, all sorts of exotic forms of energy and matter existed, such as quark matter, strange matter, and even antimatter, existed in great abundance. The universe continues to expand and cool.

<u>The formation of primordial black holes</u>

From the time that the Higgs boson activated up to now, which is roughly one microsecond or a millionth of a second after the beginning of everything, there is a tiny excess of matter particles over antimatter particles. And by tiny, I mean one extra matter particle, be it a quark, lepton, or even a boson, in a cosmic ocean of trillions possibly quadrillions of matter antimatter pairs, that are constantly being created and annhililated. The universe continues to expand and cool.

There shoudn't be a difference at all. Zero, not one in anything, but zero. The fact that the universe went out of equilibrium at all is the question. How did the cosmic balance get violated? To answer this, we need to think about a series of steps that

occurred throughout this time period. And it all begins when the annhililation rate exceeds the production rate of matter antimatter pairs, such as when the heaviest exotic particles annhililated away into less heavy particles, and so on. With only the lightest tier of particles present at this point in history, means most of the energy of the annhililation goes into heating the lighter particles by a given fraction. The universe continues to expand and cool.

As it turns out, this extra heat creates turbulence in the plasma of annihiliating particle antiparticle pairs. And this turbulence usually balances out given enough radiation pressure. But whenever we lose further particle species, their isn't quite enough pressure for the plasma to even out. This means that there are moments in the history of our universe from 10 picoseconds onwards to 1 microsecond and beyond, that are going to have turbulence throughout. The universe continues to expand and cool.

When such turbulence persists under the constant struggle of gravity, small pockets of the quark gluon plasma collapse under their own weight, forming primordial black holes with the mass of a mountain at first. Then as the next tier of particles annihilate away, more turbulence is created, and plenty more prmordial black holes are created. These black holes have more mass, about that of a whole planet or large moon. But back then in the first microsecond, it was only a raging firestorm of equal amounts of matter and antimatter. The universe continues to expand and cool.

The creation of new black holes follows when the density overcomes the expansion rate of the observable universe. The observable universe wasn't billions of light years across like it would be by the time this text you are reading was written, no. At 1 microsecond, the size of a universe that would be known would just be the time that information travels in that time. The fastest rate, which is the speed of light in a vacuum, makes the horizon of any observable part of the universe only 300 meters or 1000 feet in radius. Such a black hole would be the mass of a large planet to that of a small star in the modern universe. Inside such a contracting bubble, an event horizon forms, that makes the traversal one way, and a black hole is formed. In any case, the plasma inside appears to go through a big crunch, because there is no knowledge of continued expansionn outside the cosmological horizon. The universe continues to expand and cool.

But with such black holes forming all over the universe, creates huge explosions, at the scale of supernovas or larger, that create enough disequilibrium to force the imbalance between matter and antimatter. At this point in time, so far not much matter has been created from what seems like the vacuum. But the universe is not a vacuum, its a quark gluon plasma, that flows like the best motor oil, except its temperature is plummenting in from the quadrillions to trillions of degrees. These black holes where what created the first imbalance, that as we shall soon see, will form itself into all of the nuclear matter that will ever exist. The universe continues to expand and cool.

The quarks bind together into hadrons

Then, all of a sudden, right after 30 microseconds, the quarks and antiquarks bind together, into subatomic particles known as hadrons. Hadrons made of three matter quarks are known as baryons, and hadrons made of a quark antiquark pair are known as mesons. The lightest baryons are the nucleons, which have a mass of 940 MeV. The lightest mesons are the pions, which have a mass of 135 MeV. However the up quarks with masses of 2.01 MeV, and the down quarks with masses of 4.79 MeV, that make up these subatomic particles only contribute 1% of the total mass of the hadron. Strange quarks are the only exception to this rule, as they have a mass of approximately 95 MeV. These hadrons are confined, as it is impossible to remove any quarks or gluons for that matter, without creating other hadrons in the process. The fundamental strong nuclear force only acts within one of these subatomic particles, because of this confinement. This confinement is caused by the taffy life gluon flux tubes that emerge from the center of the hadron, and stretch out to the locations of the quarks. Again this is caused by the self interaction of the 8 types of gluons. Most hadrons have a physical size of around 1 femtometer. This is 100 times larger than the weak force can carry. In terms of electron volts, this corresponds to a value of 197 MeV. The temperature is dropping below 200 MeV. The universe continues to expand and cool.

Over the course of a few dozen microseconds after the quark gluon plasma transitions to a baryon meson gas, the resulting subatomic particles are much to heavy to exist, and as such they annhililate away. The small excess of matter quarks over antimatter quarks that are left behind, is now in the form of nucleons, which are the protons and neutrons. This is the very first real tangible matter in the universe. A proton is made of 2 up quarks and 1 down quark, and a neutron is made of 2 down quarks and 1 up quark. Yet these nucleons are outnumbered in the billions by the rediculous numbers of pions, that are being copiously produced in the seething 150 MeV heat. But what is important here is that as all kinds of quarks and all kinds of antimatter quarks, warred for supremacy, emerged a thin excess of protons and neutrons: The particles of matter we know today. The universe continues to expand and cool.

Yet the temperature not going to be this hot for long. So as the temperature drops below 135 MeV, pions begin to decay. The pions being made of quark antiquark pairs are very unstable. The charged pions are made of one up quark, and one anti down quark, or vise versa, and thus take a couple dozen nanoseconds to decay. The neutral pions on the other hand, are made of a quark antiquark pair of the same quark flavor, and so annhililate almost immidiately. Muons which weigh around 105 MeV, go next, after a couple microseconds. The universe continues to expand and cool.

And finally we reach the 100 microsecond mark in the history of the universe. If we were to take a census of all the particles in the universe as of now, there are 45 billion neutrinos, 20 billion photons, 15 billion electrons, 15 billion positrons, and finally 8 protons and 8 neutrons. There is also an excess of 8 electrons, that meet the

requirement of the universe being electrically neutral. Like the W and Z bosons, the charged and neutral pions, act as short ranged force carriers. Since the pions are hundreds of times lighter than the electroweak bosons, they carry the residual nuclear force out to a 2.5 femtometers of distance, which is around three nucleons wide. However, it is still way too hot for the first nuclei to form. Yet. The universe continues to expand and cool.

The formation of primordial black holes creates our material universe

Once the quark gluon plasma has transitioned to a baryon meson gas, there was much more annihilation going on than ever before. And as we know, this leads to the formation of black holes. Since the observable part of the universe extends for about 2 miles, or 3 kilometers, means that the black holes that formed have that size, with a mass of about our home star, the sun, in the modern universe. But back in this moment when the plethora of Kaons, Lambdas, Omegas, all annhililated, there was so much turbulence, that not only did huge solar mass black holes form, but because of the massive amount of heat generated by the annihilations, pockets of the baryon meson gas, surrounding newly formed primordial black holes, returned back to the quark gluon plasma that existed prior. The universe continues to expand and cool.

When this plasma returns to the hadronic state once more, it triggers more annhililations, and thus more black holes. This feedback loop created the bulk of black holes that will ever exist! We call this substratum of primordial stellar mass black holes, dark matter, which as the modern universe matures, will form galaxies, stars, the whole shebang. But back in the first instants of creation, none of this structure exists, except for the matter particles that will form it. Right now the total abundance went from one in quadrillions of extra matter to one in a few billion of extra matter particles. The universe continues to expand and cool.

That is like saying we had a tiny amount, now we have a surplus of extra protons and neutrons, that will be enough to form the entire material budget of existence. Of the mass of dark matter to baryonic matter, 84% is invisible primodial black holes, and 16% is the thin excess of protons and neutrons! This will be slightly modified in a few hours to a few days from creation, but here is the dominant source. The universe continues to expand and cool.

From here on out, the black holes spread out as the universe expands, some black holes merge, releasing gravitational waves, but as the dominant form of matter has now arrived, the question is no longer, about what the imbalance is, but what will happen to this thin excess of baryons created in this apocalypse. The universe continues to expand and cool.

The matter antimatter annihilation

But going back to a fraction of a thousandth of a second since the beginning, for every proton or neutron created in this cosmic game of pinball, billions of photons, of gamma-ray light, were also created! The universe was dominated by energy, and yet again, what I referred to as a thin excess will amount to all of the matter in our universe that will ever exist! Along with the billions of gamma rays, there also exist billions of electrons and antimatter electrons, also known as positrons. The temperature is still too high for electron-positron pairs to begin annihilating, and they remain in flux. The universe continues to expand and cool.

Now we reach the 1 millisecond mark. The temperature is in the tens of MeV. Electrons and positrons also known as antielectrons, even the sea of photons and neutrinos, continue to bash into the few isolated protons and neutrons, turning them into one another. This happens due to the weak nuclear force that acts at a scale 100 times smaller than a proton or neutron. When an electron gets near an up quark inside a proton, of which the quark has an electric charge of +2/3. If it is close enough, the up quark will transmute into a down quark, with a charge of -1/3. The proton has turned into a neutron. If this neutron hits a positron, it will convert back into a proton. The massless neutrinos, that act exclusively with the short ranged weak force, also have the ability to convert protons and neutrons into eachother. So at this point in time, protons and neutrons are always converting back and forth. So at first, the firestorm of mostly gamma radiation and electron-positron pairs, contains an equal number of protons and neutrons. The universe continues to expand and cool.

However the neutron is only 780 keV heavier than a proton, and the electrons weigh in at a tiny 511 keV. So as the temperature drops to a few MeV, it becomes much easier to make protons than to make neutrons in these conversions, and so there are fewer and fewer neutrons around as compared to protons. Over the next second or so, electrons and positrons are also starting to go through the same annihilation, that the quarks went through right at the start of creation. Then, as we pass the 1.8 second mark, the electron positron pairs begin annhililating, and over the next few hours, the concept of antimatter will become a distant memory. But now, with a universe 3 seconds old, some lower energy electrons and positrons still exist. The universe continues to expand and cool.

Also around this time, the neutrinos have not annhililated with their antineutrinos, because of their almost massless nature. Yet, as the universe continues expanding as it always does, the neutrinos and antineutrinos are having great difficulty turning protons and neutrons into one another. This is because of the extremely short range of the weak force. When this process begins, the ratio of number of neutrons to number of protons, was dropping dramatically for a few seconds. The universe continues to expand and cool.

So now that neutrinos cannot change the number of protons or neutrons, the declining neutron number then settles at a value of 3 neutrons per 13 protons. Yet, neutrons are ever so slightly heavier than protons, and should decay. As of now the

lifetime of a neutron is around 880 seconds, which is too far into the future as of now. The universe continues to expand and cool.

One thing to note is that when the majority of neutrinos stopped interconverting the protons and neutrons, there were still quite a few higher energy neutrinos in the distribution, still in equilibrium with the remaining electrons, positrons, and photons. This means that for a few more seconds, these neutrino outliers are still interconverting the protons and neutrons between one another, so while the number 3 neutrons per 13 protons seems like it should be taken at face value, it isn't set in stone. It is a complicated mixture of particles that interacts based on the temperature and concentration of a given sub-species of particles after decoupling. This is the way things are for most gradual proccesses. However after 10 seconds, this cosmic neutrinos background will be essentially non-interacting with the rest of matter, except through the gentle tug of gravity, which "sees" all forms of energy and momentum present, but not their structural content. The universe continues to expand and cool.

<u>The formation of galactic black holes</u>

After a second since everything began, more and more massive black holes have been forming. With a radius of over 1 light second, any black holes formed from here on out, will have masses of over a 100 thousand suns. Compare this to the first few microseconds, when black holes produced the dominant forms of matter in our universe, But since the universe has been expanding and cooling, the density of matter and radiation is too thin for more matter production, despite the fact that there are billions of electron positron pairs to yet annhililate. The universe continues to expand and cool.

So, black holes are still forming, just at a slower rate. However, they are not producing much matter at this point from the annhililating electrons and positrons. Trace to insignificant amounts were produced. At this point, the black holes are supermassive when they from. Supermassive's are the seeds for the later formation of galaxies. So to summarize galaxies required all three stages of primordial black holes to form. The universe continues to expand and cool.

Why? The dark matter that will hold the galaxies together into the cosmic web, is already forming, and being that it is the combined gravity of all black holes, microscopic, stellar, and supermassive, without them structure wouldn't exist. Not to mention that stellar mass primordial black holes formed the excess of matter over antimatter, and now constitute the bulk of dark matter as of now. Even the supermassive's are what is needed to create the galaxies themselves. The universe continues to expand and cool.

So to summarize a galaxy is made of both dark matter and normal baryonic matter, that formed in the first microsecond, and needs the supermassives to spark them into life. Without any stage of matter or black hole formation, galaxy formation would be impossible! The universe continues to expand and cool.

As soon as the supermassive black holes have formed, all three stages of black holes begin interacting with one another through gravity. The gravitational waves produced from the mutual gravity of passing black holes stirs the plasma ever so slightly. Also the black holes are immune to the radiation pressure, so they're structure immediately roughens to form a cosmic scaffolding of purely black hole dark matter. The universe continues to expand and cool.

The nuclear fusion into alpha particles

One second passes, then five, then 20 seconds have elapsed. Then a minute passes. The residual nuclear force, is strong enough to bind the nucleons together, but this isn't happening at all. This is because when a proton and neutron collide and form a deuteron, a high energy photon will break the first nuclei right back down to the nucleons that already exist. But a deuteron has a binding energy of 2 MeV, so it should be stable enough to exist. Nope. Remember that there exist 20 billion photons. Even if the average temperature of this thermal radiation is on the order of 100 keV, or 0.1 MeV, there will still be a few outliers with energies over 2 MeV. Even if a few very high energy photons exist, its over for a deuteron. The universe continues to expand and cool.

Yet again, as always, the universe expands and cools. Also, neutrons decay, so given that we started with 3 neutrons at first, after 3 minutes, there are only 2 neutrons left, as one of the neutrons has beta decayed back to a proton, along with an electron and an antineutrino. We now have 20 billion gamma ray photons, along with 14 protons and 2 neutrons. Also, an excess of 14 electrons also exist, along with the hundreds of millions of electron-positron pairs that are under constant annhililation. The universe continues to expand and cool.

So this means that for the next four minutes or so, not much changes to the firestorm of gamma-rays and annihilating electron-positron pairs. A few of the many neutrons that still exist, have naturally decayed back into protons, so while the neutron to proton ratio used to be 1 in 5 a few minutes ago, right now its closer to 1 in 7. The average lifetime of a free neutron is 880 seconds, otherwise put, 14 minutes and 40 seconds. I should note that although the temperature is well high enough for fusion to occur, the sheer energy and number of gamma ray photons will shred down any nuclei that try to form, so the universe, as of now, contains only free protons and free neutrons. The universe continues to expand and cool.

Then, right after the clock strikes 4 minutes and 20 seconds, does something dramatic happen: Nuclear fusion! Now, as the temperature decreases, there are less and less photons with energies of 2 MeV, and when the temperature, or average photon energy, drops below 78 keV, deuterons are stable, and the fusion process continues on. In other words, the intensity of gamma radiation is now low enough for the first nuclei to not be blown apart. Most nuclei that could form at this point, like tritons or helions, also have binding energies not much more than deuterons. However there is one special

kind of nucleus, known as an alpha particle. It has a whopping binding energy of 28.3 MeV. That is to say 28,300 keV! The universe continues to expand and cool.

At this point the majority of the nucleon excess is in protons, alpha particles, and neutrons. But if one zooms out to get a larger sample size, not of 16 nucleons, but of 10 million nucleons, one finds out that not only do other nuclei form, but their abundance shows they act as intermediate steps from the conversion of single nucleons into alpha particles. Out of 10 million nucleons, the peak number of nuclei is the following: 5,041 deuterons, and 683 tritons. A deuteron consists of a bound state of one proton and one neutron. A triton consists of a bound state of one proton and two neutrons. After this peak in abundance, the deuterons and tritons get used up in the formation of more alpha particles. The universe continues to expand and cool.

An alpha particle consists of two protons and two neutrons. The stability of alpha particles, is extremely important in the near future of the nuclear reactions. Every proton and every neutron in existence are now smashing together to build alpha particles: Clusters of 2 and 2, that is to say, 2 neutrons with 2 protons. The universe is now 265 seconds, or 4 minutes and 25 seconds old. At this point, as soon as alpha particles form, it becomes extremely difficult to destroy them. All nuclei that contain any neutrons, including the soon to decay neutrons themselves eventually get locked up inside these alpha particles. The universe continues to expand and cool.

As the number of free neutrons declines to zero, the nuclear reactions which have been proceeding rapidly, radically changes its form. Out of 10 million nucleons total, the following abundances have occurred as follows: The number of deuterons drops from 5,041 deuterons to 264 deuterons. The number of helions grows from zero to 102 helions. A helion consists of a bound state of two protons and one neutron. Also, the number of tritons drops from 683 tritons to 1 triton nucleus. This it will remain for years to come. The universe continues to expand and cool.

For the next 10 seconds, the number of alpha particles, grows to a value of 1 alpha particle for every 12 protons. The story of alpha particles, is not over yet. As the clock reaches 4 minutes and 30 seconds, all of the neutrons get used up in the reaction, and the 10 seconds of nuclear fusion grinds to an immediate halt. Why so early? Even though fusion is still occurring in vast quantities, not much happens to the composition of the nuclei. The 12 protons, and 1 alpha particle, are all smashing into one another. When such a collision takes place, it produces an unstable nucleus with 5 or 8 nucleons. Such a nucleus will undergo alpha decay, and revert right back to the same nuclei as before. The universe continues to expand and cool.

Also around this time, from 5 to 20 minutes after the big bang, there is still plenty of fusion occuring, but just because heavier elements are improbable doesn't mean they do not form. So to be fair, I will list out the trace nuclei that do form around this time. We talked about 12 protons and 1 alpha particle, that dominates the nuclear budget. Then we expanded our inventory from 16 nucleons up to 10 million, and found

other nuclei, namely 264 deuterons, 102 helions, and 1 triton. Tritium has a lifetime of 12.32 years which is too far into the future as of now. But out of 1 quadrillion nucleons or 15 powers of ten less than hydrogen and helium totals to, there are the beryllium and lithium isotopes. In order of declining abundance, we have the unstable beryllium-7 nuclei with a lifetime of 53.22 days, that are currently decaying into lithium-7 nuclei. Currently Beryllium-7 is at an abundance of one nucleus in 500 million nuclei total. In the first 5 to 7 minutes after the beginning, the abundances of Lithium-7 peaked at 1 nucleus in 20 million nuclei total. But after 10 minutes, there was still plenty of nuclear reactions that destroyed Lithium-7 so it dropped to a value of 1 nucleus in 3 billion nuclei total, with Beryllium-7 now in the lead of trace isotopes. Then there is Lithium-6, with an abundance of 1 nucleus in 1 trillion nuclei total, making it the rarest of trace isotopes. The content of the nuclei are the following: Beryllium-7 has 4 protons and 3 neutrons, Lithium-7 has 3 protons and 4 neutrons, and Lithium-6 has 3 protons and 3 neutrons. All of these trace nuclei are easy to destroy and hard to make, hence the very low abundances. The universe continues to expand and cool.

Strangely however, free neutrons, which were common before the universe was 4 minutes old, are now not completely gone! Yes it is true that the alpha particles contain neutrons, so when that majority forms and stabilizes, the number of free neutrons declined rapidly. But now, 5 to 20 minutes after creation, the neutron number doesn't quite settle, not because of nuclear reactions, but because all neutrons have an average lifetime of 880 seconds which is 14 minutes and 40 seconds. So although not zero, it is still dropping. At 8 minutes, there is one free neutron per 35 million nuclei total. So while protons and alpha particles domainate the nuclear matter budget, neutrons are one of the trace nuclei, but are extremely radioactive and after 1 hour or so, will be essentially all gone, converted back to stay as protons one last time. The universe continues to expand and cool.

After some more time for the universe to expand and cool, the universe ages past the 20 minute mark. However, these stable nuclei are positively charged, and like charges repel. This means that there is an electric potential barrier right beyond the reach of the residual nuclear force. As the nuclei slow down, there is less and less kinetic energy available, and by the end of the first hour of the universe, almost every nucleus avoids every other nucleus around it. The ever expanding and cooling universe now has a chemical formula of 12 protons, 1 alpha particle, which goes along with an excess of 14 electrons over millions of electron-positron pairs, and also the 22 billion photons of gamma-ray light. The universe continues to expand and cool. The fusion process halts completely, as the 12 protons, and 1 alpha particle, not only fail to create new nuclei, but at this point they don't touch eachother in the first place. The nuclei just avoid eachother from now on. The universe continues to expand and cool.

In the background, the invisible intangible radiation of the 18 billion neutrinos has been existing alongside the 22 billion gamma ray photons. Since neutrinos decoupled

within the first second of existence, they've been hardly interacting with anything. But that doesn't mean they do not exist. As electron-positron pairs annhililate away into photons, this increases the photon temperature, but not the neutrino temperature. And since everything coupled to the photon temperature as of now, its useful to ignore the lower but original neutrino temperature. If you must know, compared to a photon temperature of 1, the neutrino temperature is the cube root of 4/11. The universe continues to expand and cool.

Mentioning the average temperature is fine if everything were in perfect thermal equalibrium. Almost so, but the fusion of protons and neutrons into alpha particles produced plenty of high energy gamma rays, on the order of 1 to 10 MeV. Since fusion has already occurred and halted, the average mjority photon temperature is much lower. It will take a long time for these high energy photons to equilibriate with the other photons to reform the perfect thermal distribution. Compared to neutrinos in the first seconds, they also have high energy outliers, but neutrinos don't interact with matter, so those outliers still exist, stretched out by the expansion of space itself. But we should expect that after a few months the equilibrium of photons, which do interact with matter such as electrons and nuclei, will eventually equilibriate once again. The universe continues to expand and cool.

Hours into the creation of the universe, the last few dozen electron-positron pairs finally annihilate away into even more photons of gamma-ray light. The particle count is now, 12 protons, 1 alpha particle, 14 electrons, and 22 billion photons. This it will remain for almost an eternity. As a sense of comparison, a proton weighs as much as 1,836 electrons, and an alpha particle weighs as much as 7,294 electrons. The universe continues to expand and cool.

<u>The black holes arrange into a cosmic web</u>

At this point, we are hours into the formation of everything. No more annhilations are happening. By remembering that the universe is radiation dominated, means a few things. First, there are no more black holes that are to form any more. The past annhililations were purely thermal in nature, and are classified as radiation. With only the ghostly neutrinos and the photons of gamma ray and X ray light dominating the energy budget for radiation, no annhililations are occuring, and as such the formation of primordial black holes has stopped. The universe continues to expand and cool.

The second effect of the radiation dominating, is that its pressure holds up normal baryonic matter from collasping under it's own gravity. And also, it's the photons of light doing the work of keeping the turbulence down to a minimum, as the neutrinos hardly interact with anything. From now on, this primordial plasma, of protons, alpha particles, electrons, and photons, are all scattering off one another hard enough to keep the plasma uniform to 1 part in 100 thousand. This it will remain for a very long time. The universe continues to expand and cool.

However, the primordial black holes that already exist don't follow this same logic. They continue to suck in everything nearby, including other black holes. Even though no new black holes have formed, the 84% of dark matter continue to exist. With the other 16% of normal matter locked up with the radiation, it is this 84% that will build the cosmic web. So although, primordial black holes and normal matter seem to be running the show, weight for weight, its the radiation energy that has dominated and will continue to domninate for a very long time, possibly for the next few to dozen millennia into the future. The universe continues to expand and cool.

But that doesn't stop the primordial black holes from pulling on one another by their own self gravity. Black holes orbit and merge, releasing gravitational waves. Soon, we have a cosmic web of dark matter, that although insignificant at this moment, will play a key role later on in the history of everything. The universe continues to expand and cool.

Remember those trace nuclei, like beryllium-7, lithium-7, lithium-6, and so on. They are easy to destroy. These primordial black holes also had a huge impact on the destruction of these low binding energy nuclei. Unlike the alpha particle which is a nuclear brick. The trace nuclei are like nuclear fluff. So when this radioactive nuclear mixture nears a black hole, the black holes also grow from consuming anything that cross their event horzons: all forms of radiation, all forms of matter, even other black holes. This heats the surrounding matter up in the immidiate vicinity of their accretion disks. This heat then destroys many trace nuclei in the region. This is why the total number of all trace nuclei are dropping. It isn't radioactivity. Only neutrons, which are non-existent, Beryllium-7 which is next to go, and Tritium which is to go after that. But radioactivity is passive, it is disctinct from the active fission of nuclei by high energy photons of radiation. So depending on the stability, and closeness to these hot spots, many nuclei are disintregated. This changes the abundaces of many trace isotopes in the early universe. But as time marches on, most of the accretion dies down, due to matter and radiation thinning out. The universe continues to expand and cool.

Also these hot spots are creating even more high energy photons, that while many disintegrate nuclei, the scattered light still has much to high energy, so along with the nuclear reactions that occurred from 3 to 20 minutes after the big bang, the combined effect is to make it very difficult for the photons to reach equilibrium again. But as time passes, the universe expands. The expansion of space stretches all the field quanta out, and this lowers the temperature of the universe, coupled or uncoupled. With all this time on our hands, the black holes have spread out enough to make the photons begin to reach equilibrium. But the completion of this proccess into pure random heat will take many months. The universe continues to expand and cool.

<u>The primordial plasma clears up</u>

This whole time, electrons and photons, pinball one another. The scattering of, electrons and photons, is fully described by the theory of QED. QED begins with the spacetime of special relativity. This spacetime is described by the O(3,1) group. The particles of QED, are representations of this symmetry group. Electrons are massive charged spinors. Photons are massless neutral vectors. Put together, they interact via a minimally coupled U(1) internal symmetry group. As of now, QED was not the main player in the history of the universe, as it was unified with other internal symmetries, into the SU(5) group. The universe continues to expand and cool.

However, now that the nuclear reactions have stopped, QED is what is left over. The electrons and photons form a plasma known as the cosmic soup. In it, electrons are known as cathode rays, and photons are known as X rays. The chemistry of the universe is pretty much constant after the first few hours have elapsed. The cosmic soup clocks in at a temperature of 1 keV. Otherwise put, 11.6 million degrees kelvin, and dropping. In fact, it's still too hot for anything tangible to exist at all. The universe continues to expand and cool.

A day passes, then a week goes on by. The only thing that changes is the expansion of the universe. Nothing has happened to the plasma. Even after a whole month of this expanding and cooling, the temperature, pressure, and density, are all surely dropping, yet nothing else has happened to the cosmic soup. Why? As compared to the nuclei, which are femtometers in size, and weigh in at many GeV, electrons are nothing in comparison. Even the photons that carry the electromagnetic force, is 137 times weaker than the other forces. So not only do the electrons need to slow down, they need to be captured by the nuclei, for the first atoms to form. The universe continues to expand and cool.

Every electron, cannot be crammed into a volume smaller than 386 femtometers, without producing antimatter. This is assumming that the cathode rays are moving near light speed. But, at this day and age, the cathode rays have much longer wavelengths than this, since they're moving much slower than light speed. The math of QED, says that the electrons must be moving 137 times slower than light speed for them to be captured. With the electrons so spread out compared to the nuclei already, the cathode rays must have wavelengths hundreds of thousands of times longer than the nuclei are big. And that is a big problem. The universe continues to expand and cool.

As weeks pass by, something is happening to the rarest of trace nuclei, as the beryllium-7 nuclei with a lifetime of 53.22 days are gradually decaying into lithium-7 nuclei, increasing the number close, but not quite to the value it had when the universe was only 5 minutes old. All of this is only significant at scales with an gigantic sample size of over 1 billion nuclei total. But in the grand scheme of things, nothing has changed to the chemistry of the uniform cosmic ocean of fire. The universe continues of expand and cool.

As months turn into years, then into decades, the gamma ray photons have stretched out so much, they are now X-ray photons. As the firestorm of a universe continues to expand and cool, not much happens for a very long time. The universe as a firestorm that is uniformly spread and infinite in extent, continues to expand and cool. However, if one looks at our huge sample size of 10 million baryons, something did just happen. Our last triton nucleus has beta decayed into one more helion nucleus. So while 102 helions used to exist, the extra helion added in, gives us 103 helions. This is the last time a beta decay happens for a very long time. The neutron inside the triton nucleus took 12.32 years to decay, compared to a free neutron decaying in 880 seconds, its an eternity. The universe continues to expand and cool.

After all of this radioactivity, nuclear fusion, black hole production and growth, the photons of light are slightly off equilibrium, and now the universe has produced enough of a disturbance of one part in 100 thousand. Now after many years, the composition and energy distribution of the matter and radiation has reached equilibrium once more. But instead of this heating just spreading out, it rings like a bell, or better yet, like an expanding balloon, vibrating once every few years or so. The universe continues to expand and cool.

Decades pass by, then centuries, and eventually millennia pass by in the hundreds. The universe is so much more diluted now that its self gravity, which is slowing down the expansion of the universe, takes more time to expand by the same amount as usual. Up to this point, the temperature-time relation has been following an inverse square root curve. At one second the temperature was 1.27 MeV, so at 100 seconds or 1 minute, 40 seconds, the temperature drops ten fold to 127 keV. To arrive at the temperature we take the time to an exponent of negative one half. This pattern is about to break, as the exponent will shift to a value of negative two thirds. If the temperature in the near future of 100 thousand years is 620 meV, then if we project out to 1,000 times into the future of 100 million years, the temperature will drop a hundred fold to 6.2 meV. This is because matter will soon be dominating the expansion history of the universe. The universe continues to expand and cool.

By matter I mean all forms of mass including not just nuclei and electrons, but also primordial black holes that outweigh all of our baryons by a factor of 5. But that is for a later chapter. For now also realize that its not just the photons that constitute radiation, but also the ghostly neutrinos. If we just looked at the total energy budget of the universe, around this time the temperature has finally dropped so low to the point that our small excess of matter over antimatter now dominates its gravitational evolution. The universe continues to expand and cool.

Moreover, all of the photons of X-ray light, have stretched out into photons of Ultraviolet light. However, as the universe expands, a new problem surfaces. Billions of photons might have an average energy of 5 eV or 58 thousand degrees kelvin, yet the truth is that as always there are high energy outliers. Some photons have huge energies

well over 269.3 eV and as such we will have to wait until these high energy photons are gone until atoms can fully form. The universe continues to expand and cool.

Out of the total of 10 million nucleons the following happens: As the universe expands and cools below 16 thousand degrees, each and every of the 625,000 alpha particles captures an electron forming 625,000 helium-4 ions. Of the far and few between helions, each of the 103 helion nuclei also capture an electron, forming 103 helium-3 ions. The universe continues to expand and cool.

Next, as the universe expands and cools below 7 thousand degrees, each and every of the 625,000 helium-4 ions capture another electron, this time forming 625,000 helium-4 atoms. Again, the same thing occurs for 103 helium-3 ions, as they too capture their second electrons, forming 103 helium-3 atoms. The universe continues to expand and cool.

Then, as the universe expands and cools below 4 thousand degrees, each and every of the 7,500,000 protons captures an electron, forming 7,500,000 hydrogen-1 atoms. Of the far and few between deuterons, each of the 264 deuteron nuclei also each capture an electron, forming 264 hydrogen-2 atoms. The universe continues to expand and cool.

After that, as the universe expands and cools below 3 thousand degrees, the number of free protons and free electrons have not fully recombined into hydrogen-1 atoms. Instead, 578 of the 7.5 million free protons and with them 578 free electrons also remain. The last 578 free electrons have for the first time in history, stopped scattering light, leaving the now visible light photons to begin streaming freely throughout the cosmos. Let There Be Light!

<u>The cosmic dark ages</u>

The universe continues to expand and cool. 377 millennia have elapsed since the beginning, and is flooded with atomic hydrogen-1 and atomic helium-4. The color temperature of the radiation is pushing 3 thousand degrees Kelvin, but is gradually dropping as we speak. Of the 10 million nucleon total. We have the following abundances for baryonic matter: 7.5 million hydrogen-1 atoms, 625 thousand helium-4 atoms, 578 free protons, 578 free electrons, 264 hydrogen-2 atoms, 103 helium-3 atoms. However it remains to be seen what kinds of chemistry will occur in this white hot, near vacuum atmosphere. The universe continues to expand and cool.

From 3,000 degrees kelvin, or roughly 5,000 degrees Fahrenheit, the radiation temperature and matter temperature though decoupled, are dropping in tandem as we speak. As the temperature drops below 1,150 degrees kelvin, or 1,610 degrees Fahrenheit, of the 10 million nucleon total, 4 hydrogen-1 atoms stick together in pairs, forming 2 hydrogen-1 molecules. The catalyst here is the rare but intermediate formation of the hydrogen molecular cation, formed of two protons held together by a single electron. It's the result of the collision between a hydrogen-1 atom, with a free proton. The universe continues to expand and cool.

Then, as the radiation temperature drops below 530 degrees kelvin, or a relatively hot 490 degrees Fahrenheit, the matter temperature begins cooling twice as fast. Instead of the radiation temperature which as a time-temperature exponent of negative two thirds, the matter temperature now has a time-temperature exponent of negative four thirds, which means that if the radiation temperature drops from 530 kelvin to a bone cold 53 kelvin, the matter temperature will have dropped to 5.3 kelvin above absolute zero, twice as cold as the radiation temperature has dropped. The universe continues to expand and cool.

Next, as the radiation temperature drops below 300 degrees kelvin, or a luke warm 80 degrees Fahrenheit, another 34 hydrogen-1 atoms have stuck together in pairs, forming an additional 17 hydrogen-1 molecules, making a total of 19 hydrogen-1 molecules. This time the catalyst here is different. It is the rare but intermediate formation of the hydrogen anion, formed of a single proton holding two electrons. This time, it is the result of the collision between a hydrogen-1 atom, with a free electron. The universe continues to expand and cool.

The 19 hydrogen-1 molecules are all each formed of two protons held together by two electrons. Below room temperature, the following census holds in detail: 7,500,000 hydrogen-1 atoms, 625,000 helium-4 atoms, 578 free protons, 578 free electrons, 264 hydrogen-2 atoms, 103 helium-3 atoms, and our 19 hydrogen-1 molecules. The universe continues to expand and cool.

So far, we have been talking about only a small fraction of what matter and energy exists out there. It is like saying we have a jar filled with 100 grams of gas. As we know from earlier, 75 grams would be atomic hydrogen-1, and 25 grams would be atomic helium-4. The truth that has been overlooked is this conundrum: An additional 530 grams of invisible intangible matter also exists.

This form of matter consists of countless primordial black holes. A black hole is a region of space with a gravitational pull that is so strong that not even light, the fastest thing in the universe, cannot escape from its deadly grip. This is an extremely important concept: All fundamental particles are rotating black holes with the same mass and same spin as the wave they constitute. This means that all black holes are fundamental waves too in the same sense as of any other wave, such as the electrons of electricity and the photons of light are all fundamental waves. This means that we should widen our search for extremely exotic wave types that hardly interact with anything. These primordial black holes will be a fundamental turning point in the evolution of what follows.

The fate of the universe rests on the unification of physical law. There are three main sets of new phenomena, that were incorporated into the three pillars of modern science. The three pillars are: special relativity, universal gravitation, and quantum mechanics. The associated physics reduces to our common sense, by making a series of approximations. So even though our common sense worldview works well in the everyday, in the grand scheme of things, our universe is stranger than fiction.

With this in mind, a black hole is simply a natural consequence of these three main concepts of physics. In a sense, every black hole is just as fundamental as every other particle of matter and light that exists. In fact, the only difference is their mass and spin. Therefore, all rotating black holes are just as "real" as anything else that exists. Lets see what has happened to this mysterious form of matter, so that we can continue our story of everything!

All of these black holes have been colliding and merging, since the beginning, forming a sprawling network of black holes, called the cosmic web. The cosmic web is truly massive. But the reason I bring this up now is because, before the universe was 377 thousand years old, the radiation pressure was holding up the ordinary matter from gravitational collapse. But now, going back to when the light has been set free, there is no more radiation pressure to prevent collapse, and collapse it goes. The universe continues to expand and cool.

As the universe expands and cools, over the next few million years, the color temperature dropped from white, to pale yellow, to vivid orange, to cherry red, and finally fades into the near infrared. If we were somehow around back then, we could feel these rays as radiant heat, although there would be nothing to see. Just a uniform still darkness all around. The universe continues to expand and cool.

As for the matter, the gas is no longer uniform, and as the contrast increases, the gravity of both the black holes and the atoms of hydrogen and helium, becomes unstoppable. In just a few hundred million years, the cosmic web of gas starts resembling the cosmic web of black holes. This is like the scaffolding of a building. The black holes are the scaffolding, and the gas of normal matter is following the same pattern. The universe continues to expand and cool.

As this new consolidated cosmic web continues to expand and grow, the scene will soon change dramatically to include new structures never seen before! The cosmos is still cold and darkness envelops the night sky. But a chain of events will form a hierarchy of scale, where everything will eventually depend on everything else. The universe continues to expand and cool.

But before the sky lights up, the darkness has one last secret. Of the rarest nuclei that exist, atoms of lithium stick together in rows and columns to form dust grains, and are very rare. Each atom is one in trillions now, but the last time we counted the census it was rounded to 1 part in 16, 1 part in 10 million, and 1 part in a quadrillion. Within this last quadrillionth of matter, rare isotopes like carbon, nitrogen, oxygen, and many others, have been created in the first hour, but are so rare to not contribute a signifcant difference. Could it be that life on earth arose, because there was a random surplus of these nuclei? Everywhere else, there must have been negligible amounts, and as such we will only find life where the materials will be concentrated. This will soon change as the first stars form, within galaxies: Places that stars live out their lives and die. The universe continues to expand and cool.

The formation and evolution of cosmic structure

The universe is now 300 million years old, and as the cosmic web continues to expand and grow, the first irregular galaxies form along the filaments of the cosmic web. Just like dew drops on a spider's silk, each of these galaxies is a loose collection of stars, gas, dust. As time marches on in the billions of years, the irregular galaxies merge into larger irregular galaxies. As it turns out, the densest regions of the cosmic web are where the irregular galaxies form. But since the cosmic web grows, the galaxies are carried along with the filaments of the invisible matter that consitutes most of the cosmic web. Whenever the filaments of the cosmic web lock together, is when most galaxies cluster and merge.

Zooming inside one of these irregular galaxies, there is plenty of gas from the big bang, and along with the dust from previous supernovae explosions, creates a cloud of diffuse gas known as nebula. The nebula fragments into a few collapsing clouds, which in turn forms the next generation of high mass stars: Blue main sequence stars. 10 solar masses, and their surfaces burn brightly as 20 thousand degrees in the ultraviolet, as a sky blue color. The nuclear fusion has restarted once again, this time within the cores of stars!

After another brief 100 million years later, after its formation, the blue main sequence star, runs out of its initial supply of nuclear fuel. It expands from a radius of 5 suns to 530 suns. Its surface temperature drops to 4000 degrees, or about that color of incandescent white light bulb. It is now an orange supergiant star, with its core temperature skyrocketing, inside, it has onion shells of fusing elements and isotopes. It lasts for only another 10 million years.

Every massive orange supergiant star explodes as a type II supernova! This star is no exception. The core has fused its last supply of elements into Iron-56, the most stable nucleus that exists. The core implodes in fractions of a second, into a neutron star remnant. The resulting conversion of nuclei into pure neutrons relases plenty of energy, that ricochets the rest of the stellar matter, creating a massive shockwave that blows it up. The energy from such an explosion outshines the rest of its galaxy for weeks, if not months. The explosion leaves behind a neutron star, which is a dead stellar core of compact neutron matter.

The resulting elements from type II supernovae contains: 23 carbon-12 nuclei, 5 nitrogen-14 nuclei, 156 oxygen-16 nuclei, 14 neon-20 nuclei, 2 neon-22 nuclei, 6 magnesium-24 nuclei, 4 silicon-28 nuclei, 2 sulfur-32 nuclei, and 2 iron-56 nuclei. Massive star explosion, after massive star explosion, the galaxies are being seeded with the elements needed for life itself. Elements like oxygen, which when combined with hydrogen from the big bang, forms lots and lots of water, the universal solvent. The gases and dust grains disperse into the interstellar medium.

A neutron star remnant is a very amazing object. A dense quantum object made of a plasma of only neutrons. However the neutrons are as compact as they can be. This is the same force that prevents solid matter from passing through itself, but here, it applies to the whole neutron star remnant at once: An atomic nucleus as massive as a star. For some facts, a neutron star remnant, has the mass of up to 3 suns, and a radius of a large city.

It is now almost 2 billion years after the big bang. We zoom back out to the intergalactic scale. A billion years have elapsed since the first irregular galaxies formed. As the invisible matter of the cosmic web locks together, two sheets of this invisible matter, with each sheet being a network containing more than a few dozen irregular galaxies, merge together. What was two galaxy clusters is now a grouping of a few larger galactic mergers. Gas, stars, dust, are all flung in all directions. Gravity brings the matter back together into three large spiral disk galaxies, which in turn ignites more stars within these new spiral galaxies. This creates their spiral arms and pinwheel character. In the mean time, the invisible cosmic web has grown so much by now, that what was a network of irregular galaxies is now three huge hubs of the invisible matter, inside each a spiral galaxy. It is as if a cosmic dance is taking place, with the skaters replaced by huge incandescent galaxies. The three new spiral galaxies continue to absorb more and more irregular galaxies into their spiral arms. This slightly disrupts their spiral arms, but after another few billion years, each of these spiral galaxies has grown considerably, now as three grand pinwheels.

Zooming inside one of these spiral galaxies, there is still lots of gas from the big bang, and along with even more of the dust from previous supernovae explosions, which creates several more clouds of diffuse gases that we know as nebulae. These nebulae fragment more, and due to extra turbulence from supernova explosions in nearby spiral arms, they collapse and fragment further into a few hundred collapsing clouds, which in turn forms the next generation of intermediate mass stars. One of these stars is a sun-like star! This star, and all the others in the loosely bound cluster, known as an open star cluster, burn with a surface temperature of 6 thousand degrees, which looks more or less, like white daylight. The open star cluster disapates its sun-like stars into the disk of the spiral galaxy that birthed them. Nuclear fusion has it's torch passed down to the next generation of stars!

Then, 5 billion years after its formation, this sun-like star expands to 60 times its original radius. It is now a red giant star, and has a surface temperature of 3000 degrees, about that color of an older incandescent light bulb. The star's core has begun to fuse its helium-4 ash into carbon-12 and oxygen-16. This bloated angry red giant star, lasts for another 1 billion years, until it's core becomes dangerously unstable. The core gets denser and energy waves from it, expelling plenty of stellar matter away from the diffusing red giant star into what is known as a planetary nebula. After a few thousand

years later, what is left over is a very dense dead stellar core of compact atomic matter known as a white dwarf.

The resulting planetary nebula contains: 69 carbon-12 nuclei, and 12 nitrogen-14 nuclei. Like the other supernovae explosions, the planetary nebulae from countless red giant stars seed the galaxies with even more matter, necessary for life, which includes a plenty surplus of carbon. The gases and dust grains disperse into the interstellar medium.

A white dwarf remnant is a quite amazing object. A dense quantum object made of a plasma of nuclei and electrons. However the electrons are as compact as they can be. This is the same force that prevents solid matter from passing through itself, but here, it applies to the whole white dwarf remnant at once: An atom as massive as a star. For some facts, a white dwarf remnant, has the mass of up to 1.4 suns, and a radius of a planet or moon.

Since many sun like stars can come from the same fragmenting nebula, it is very common for this generation of stars to orbit in pairs, and rarely triplets. This allows for a new phenomenon to occur. With an orbiting pair of sun like stars, one might form a planetary nebula first, before the other star even expands out to become a red giant. When the red giant's atmosphere touches the other white dwarf remnant, it captures lots of gas and dust. The nuclei slam onto its surface, and sometimes can temporarily cause nuclear explosions called novae. However, once the white dwarf remant starts to fuse elements into Iron-56, the same element that kills massive stars one by one, the remnant detonates. The resulting shockwave shreds the other star down to a cloud of plasma. This is called a type Ia supernova. The energy from such a supernova explosion outshines the rest of its galaxy for many weeks. Since every white dwarf remant always reaches the same mass before they explode, each exploding white dwarf remnant always has the same luminosity. A standard candle if you will…

The resulting type Ia supernova contains: 2 silicon-28 nuclei, 1 sulfur-32 nucleus, 1 argon-36 nucleus, and 3 iron-56 nuclei. Although iron as a nuclear fuel, is a star killer, most of the iron in the cosmos comes from supernovae like these. The gases and dust grains disperse into the interstellar medium.

It is 9 billion years since time began. We zoom back out to the intergalactic scale once again. At this point the three main spiral galaxies are about to merge together. In front of us is a barred spiral galaxy about 100 thousand light years across. 2 million light years, over to the left is a grand spiral galaxy, that is 250 thousand light years across. Another 800 thousand light years, further left, is a much smaller pinwheel galaxy, and is only 70 thousand light years across. However, as billions of years in cosmic time pass by, the two main galaxies approach one another, spinning through space, in a knife edge configuration. During their collision, they pass right through each-other, flinging their guts millions of light years into space. Meanwhile, the smaller spiral near the system's edge, shoots right through the crime scene at a relatively high speed. The seemily

double spiral galaxy with two long tails, gradually reforms itself into a new conglomeration of old stars known as an elliptical galaxy. The remants of the two tails continue to get eaten up by the newly formed elliptical galaxy's immense gravity. The smaller galaxy, comes back into the scene and is absorbed into the new huge galaxy. A slightly flattened eliptical galaxy, known as a lenticular galaxy.

The last stars to form are red dwarf stars: stars smaller than the sun, and far dimmer. Their surface temperature is around 3500 degrees, and are about a quarter of what would be our sun's mass and radius. They steadily fuse hydrogen-1 into helium-4, over a great timespan. The first of these stars will only fade after many trillions of years into the future, too far into the future as of now. Most red dwarf stars, come in pairs, triplets, or even larger solar systems. Again this is due to the fragmenting nebulae. With much more dust, and plenty of turbulence, from previous supernovae, later and later generations of stars, will tend to be smaller, and more numerous.

This whole process of gravity shaping the cosmic hiearchy, is what makes the universe such an interesting place. The cosmic web expands outwards with its filaments and knots growing. Within the web's densest interconnections, we have interacting galaxies, with each galaxy an orbiting collection of stars, gas, and black holes. Galaxies merging and evolving from small irregular lumps to great spirals and pinwheels, and lastly to huge diffuse spheres of old red dwarf stars. Deeper still, entire generations of stars have come and gone, spreading the raw ingredients for new solar systems to form later on. And in the process, we might figure out how our universe works!

Chapter 8: Concluding Remarks

We have seen the universe emerge from a theory of everything. Everything from new JWST and HST measurements of the deep sky galaxies, being seeded from primordial galactic mass black holes, to the adherence to the standard model physics, all stems from the use of the best current theories in physics. Organized by the CGH physics cube, and synthesized into the simplest theory of everything by my black hole - particle duality hypothesis. But if such a canidate theory explains everything with no more assumptions, and then gives predictions already found, the black hole duality must be confirmed.

Going through extreme scales, we found boundaries with units. Those units were actually three new laws of physics, that we unified into a picture of a single particle that we called CGH physics. We assert our black hole idea, build a theory compatible with all known physics, and look at its predictions. Now we know it is correct. Thank you Occam's razor, for making this work. Sometimes you find beauty in the symmetries of the cosmos, and this time it is in the simplicity and universality of the black hole particle duality.

Given that the theory of everything exists and is in our posession, lets now plug it into a computer, and see how it fairs…

Section 8.1: A Computer Simulation?

Such a computer simulation has to be done one way or another, and we humans are the only reliable way of doing it. We are the cosmos understanding itself, yet this undertanding of the cosmos comes at the cost of violence from dogmatic institutions, therefore we should celebrate the fact that we exist, because nature is a dangerous place. And yet, through all of us, all of this might just be a computer simulation.

- Erik Daniel Jorgensen

References and Further Reading:

Bound and Scattering States for Hydrogen in Section 3.7:
https://www.fnendzig.de/physics/HydrogenAtom_Nendzig.pdf

Kerr Metric Functions in Cartesian Kerr-Schild Form for Section 4.1:
https://arxiv.org/pdf/0706.0622v3

The Kerr Tetrad used in Section 4.4:
https://ar5iv.labs.arxiv.org/html/2203.16252

Dirac Equation in Section 4.5:
https://web.archive.org/web/20240619185747/https://strings.ph.qmul.ac.uk/~andreas/RQM/RQMnotes2005.pdf

Deriving the Spin connection from Section 4.6:
https://ar5iv.labs.arxiv.org/html/2107.01114#A1

A reference to Kerr-Schild coordinates for Section 5.4:
http://www.fen.bilkent.edu.tr/~gurses/lorentzcovariant.pdf

The wave-function used for Section 5.5:
https://arxiv.org/pdf/1205.1992

A reference for the Standard Model used in Section 5.7:
https://arxiv.org/pdf/1209.6213v2

A result for CI Chondritic Minerals for Section 7.2
https://iopscience.iop.org/article/10.1086/375492/pdf

A possible confirmation of PBH's for Section 7.4:
https://arxiv.org/pdf/2306.03903

Simulation data results used in section 7.4:
1). https://arxiv.org/pdf/1609.04979
2). https://iopscience.iop.org/article/10.1088/0004-637X/744/2/158/pdf
3). https://arxiv.org/pdf/0906.3701v1

Appendix A: Python Scripts as a Proof of Concept

My Kerr-Schild test suite and integration step for CGH physics:

```
import pygame
import numpy as np
from scipy.linalg import expm

# Here are the massive bargmann wigner equations
# using the real majorana gamma matrices
# for a space-time algebra in the curved spacetime of
# the kerr geometry in cartesian kerr-schild coordinates
# with rotating black hole mass M and spin J equal
# to the particle mass m and spin s respectively,
# with M being a free parameter alongside the coordinates t,x,y,z,
# for a wavefunction of a given fixed spin of s*hbar,
# guiding a single relativistic bohmian trajectory,
# using a globally lorentz covariant tetrad
# e^a_%mu = delta^a_%mu + (H/2)L^aL_%mu, is chosen,
# raised and lowered by the minkowski metric eta, with signature -+++.

#Kerr spacetime geometry derivation:
#https://arxiv.org/pdf/0706.0622

#Spin connection derivation:
#https://ar5iv.labs.arxiv.org/html/2107.01114#A1
#https://arxiv.org/pdf/2107.01114

#Tetrad derivation:
#https://ar5iv.labs.arxiv.org/html/2203.16252
#https://arxiv.org/pdf/2203.16252

#Kerr-Schild coordinates derivation:
#http://www.fen.bilkent.edu.tr/~gurses/lorentzcovariant.pdf


T0 = np.array([[0,0,0,-1],[0,0,1,0],[0,-1,0,0],[1,0,0,0]])
X0 = np.array([[1,0,0,0],[0,-1,0,0],[0,0,1,0],[0,0,0,-1]])
Y0 = np.array([[0,0,0,1],[0,0,-1,0],[0,-1,0,0],[1,0,0,0]])
Z0 = np.array([[0,-1,0,0],[-1,0,0,0],[0,0,0,-1],[0,0,-1,0]])

gamma = np.zeros((4,4,4))
gamma[0,:,:] = T0
gamma[1,:,:] = X0
gamma[2,:,:] = Y0
gamma[3,:,:] = Z0

def LorentzTransform(Kx,Ky,Kz,Jx,Jy,Jz):
 # Define general Lorentz Transformation matrix
 M = np.array([[0,Kx,Ky,Kz],
 [Kx,0,-Jz,Jy],
 [Ky,Jz,0,-Jx],
 [Kz,-Jy,Jx,0]])
 # Compute Matrix Exponential of Generating Matrix
 Lorentz = expm(M)
 return Lorentz
```

```
def MinkowskiMetric():
    # The spacetime invariant interval
    N = np.zeros((4,4))
    N[0,0] = -1
    N[1,1] = 1
    N[2,2] = 1
    N[3,3] = 1
    return N

def radius(a,x,y,z):
    R2 = x*x + y*y + z*z
    Ra = R2 - a*a
    AZ = a*a*z*z
    b3 = np.sqrt(Ra*Ra + 4*AZ)
    r2 = (Ra + b3)/2
    r = np.sqrt(r2)
    return r

def Potential(m,a,r,z):
    AZ = a * a * z * z
    H1 = 2 * m * r * r * r
    H2 = r * r * r * r + AZ
    H = H1 / H2
    return H

def Vector(a,r,x,y,z):
    L = np.zeros(4)
    L[0] = 1
    L[1] = (r * x + a * y) / (r * r + a * a)
    L[2] = (r * y - a * x) / (r * r + a * a)
    L[3] = z / r
    return L
def Tetrad(m, a, t, x, y, z, Kx, Ky, Kz, Jx, Jy, Jz):
    X0 = np.array([t, x, y, z])
    Λ = LorentzTransform(Kx, Ky, Kz, Jx, Jy, Jz)  # Λ^μ{}_a
    X = np.linalg.solve(Λ, X0)                    # back to Kerr–Schild Cartesian

    eta = MinkowskiMetric()
    x0, y0, z0 = X[1], X[2], X[3]

    r = radius(a, x0, y0, z0)
    H = Potential(m, a, r, z0)
    L = Vector(a, r, x0, y0, z0)                  # L^μ

    # lower index: l_μ = η_{μν} L^ν
    l = eta @ L

    # transform to Lorentz frame index: L0_a = Λ^μ{}_a l_μ = (Λ.T @ l)_a
    L0 = Λ.T @ l

    # tetrad: e_μ{}^a = Λ_μ{}^a + (H/2) L_μ L0^a
    e0 = Λ.copy()
    e0 += 0.5 * H * np.outer(L, L0)

    return e0  # shape (4,4): e[μ,a]

def KerrMetric(m, a, t, x, y, z, Kx, Ky, Kz, Jx, Jy, Jz):
    e0 = Tetrad(m, a, t, x, y, z, Kx, Ky, Kz, Jx, Jy, Jz)  # e[μ,a]
```

```
    eta = MinkowskiMetric()                          # η[a,b]
    g = np.einsum('ab,ua,vb->uv', eta, e0, e0)
    return g


diff0 = 1e-9

def Connection(m, a, t, x, y, z, Kx, Ky, Kz, Jx, Jy, Jz):
    d = diff0
    dt = dx = dy = dz = d

    # metric at the point
    G = KerrMetric(m, a, t, x, y, z, Kx, Ky, Kz, Jx, Jy, Jz)
    g = np.linalg.inv(G)

    # metrics at shifted points
    Gt  = KerrMetric(m, a, t+dt, x,    y,    z,    Kx, Ky, Kz, Jx, Jy, Jz)
    Gt0 = KerrMetric(m, a, t-dt, x,    y,    z,    Kx, Ky, Kz, Jx, Jy, Jz)
    Gx  = KerrMetric(m, a, t,    x+dx, y,    z,    Kx, Ky, Kz, Jx, Jy, Jz)
    Gx0 = KerrMetric(m, a, t,    x-dx, y,    z,    Kx, Ky, Kz, Jx, Jy, Jz)
    Gy  = KerrMetric(m, a, t,    x,    y+dy, z,    Kx, Ky, Kz, Jx, Jy, Jz)
    Gy0 = KerrMetric(m, a, t,    x,    y-dy, z,    Kx, Ky, Kz, Jx, Jy, Jz)
    Gz  = KerrMetric(m, a, t,    x,    y,    z+dz, Kx, Ky, Kz, Jx, Jy, Jz)
    Gz0 = KerrMetric(m, a, t,    x,    y,    z-dz, Kx, Ky, Kz, Jx, Jy, Jz)

    # dG[μ,ν,ρ] = ∂_ρ g_{μν}
    dG = np.zeros((4, 4, 4))
    dG[:, :, 0] = (Gt - Gt0) / (2 * dt)
    dG[:, :, 1] = (Gx - Gx0) / (2 * dx)
    dG[:, :, 2] = (Gy - Gy0) / (2 * dy)
    dG[:, :, 3] = (Gz - Gz0) / (2 * dz)

    # B[μ,ν,σ] = ∂_μ g_{σν} + ∂_ν g_{σμ} - ∂_σ g_{μν}
    B = np.zeros((4, 4, 4))
    for mu in range(4):
        for nu in range(4):
            for sigma in range(4):
                B[mu, nu, sigma] = (
                    dG[sigma, nu, mu] +
                    dG[sigma, mu, nu] -
                    dG[mu, nu, sigma]
                )

    # Γ^ρ_{μν} = 1/2 g^{ρσ} B[μ,ν,σ]
    Gamma = 0.5 * np.einsum('rs,mns->rmn', g, B)

    return Gamma  # shape (4,4,4): Γ[ρ,μ,ν]

def CurvedGammas(m,a,t,x,y,z,Kx,Ky,Kz,Jx,Jy,Jz):
    e = Tetrad(m,a,t,x,y,z,Kx,Ky,Kz,Jx,Jy,Jz)   # e[μ,a]
    # Gamma[μ,i,j] = e[μ,a] * gamma[a,i,j]
    Gamma = np.einsum('ma,aij->mij', e, gamma)
    return Gamma

def Mass_Term():
    Mass_gamma = np.eye(4)
    for dim0 in range(0,4):
        Mass_gamma = Mass_gamma @ gamma[dim0,:,:] # Majorana Cartesian Gammas
```

```
    return Mass_gamma

def Term(m,a,x,y,z):
    r = radius(a, x, y, z)
    H = Potential(m, a, r, z)
    L = Vector(a, r, x, y, z)
    # Construct Kerr Spacetime
    H0 = np.zeros((4, 4))
    for u in range(0, 4):
        for v in range(0, 4):
            H0[u, v] = H * L[v] * L[u] / 2
    return H0

def DerivateTerm(m, a, x, y, z):
    d = diff0
    dx = dy = dz = d

    hx  = Term(m, a,     x+dx, y,    z,    )
    hx0 = Term(m, a,     x-dx, y,    z,    )
    hy  = Term(m, a,     x,    y+dy, z,    )
    hy0 = Term(m, a,     x,    y-dy, z,    )
    hz  = Term(m, a,     x,    y,    z+dz, )
    hz0 = Term(m, a,     x,    y,    z-dz, )

    dG0 = np.zeros((4, 4, 4))
    dG0[:, :, 0] = 0
    dG0[:, :, 1] = (hx - hx0) / (2 * dx)
    dG0[:, :, 2] = (hy - hy0) / (2 * dy)
    dG0[:, :, 3] = (hz - hz0) / (2 * dz)
    return dG0

# Spin connection for Kerr-Schild tetrads (exact simplification per Alawadhi et al. 2107.01114 Appendix A1)
# (ω_μ)_ab = ∂_b e_aμ − ∂_a e_bμ    → only perturbation term contributes

def SpinConnection(m,a,x,y,z):
    h0 = DerivateTerm(m,a,x,y,z)
    Omega = np.zeros((4,4,4))
    for A in range(0,4):
        for B in range(0,4):
            for mu in range(0,4):
                Omega[A,B,mu] = (h0[A,mu,B] - h0[mu,B,A])/2
    return Omega


def Sigma(A,B):
    return (gamma[A,:,:] @ gamma[B,:,:] - gamma[B,:,:] @ gamma[A,:,:])/2


def SpinorConnection(m,a,x,y,z):
    omega = SpinConnection(m, a, x, y, z)
    Omega0 = np.zeros((4,4,4))
    for A in range(0,4):
        for B in range(0,4):
            for mu in range(0,4):
                Omega0[mu,:,:] += omega[A, B, mu] * Sigma(A, B)
    return Omega0

def DiracStep(M,x,y,z,Psi,Psi_x,Psi_y,Psi_z):
```

```
    m = M
    a = 1/(2*m)
    Gamma = CurvedGammas(m,a,0,x,y,z,0,0,0,0,0,0)
    omega = SpinorConnection(m, a, x, y, z)
    gamma_Tinv = np.linalg.inv(Gamma[0, :, :])
    Psi_t = - gamma_Tinv @ ( Mass_Term() @ (Psi * M)
    + Gamma[1, :, :] @ (Psi_x + omega[1,:,:] @ Psi)
    + Gamma[2, :, :] @ (Psi_y + omega[2,:,:] @ Psi)
    + Gamma[3, :, :] @ (Psi_z + omega[3,:,:] @ Psi)
    ) - omega[0,:,:] @ Psi
    #Psi += Psi_t
    return Psi_t

def PsiInit(N):
    Psi = np.zeros((2*N+1,2*N+1,2*N+1,4),dtype=float)
    for x in range(-N,N+1):
        for y in range(-N,N+1):
            for z in range(-N,N+1):
                Psi[x+N,y+N,z+N,:] = [0,0,0,0]
# WIP on my black hole-particle duality hypothesis
print(DiracStep(13,3,4,12,np.array([0.1,0,0,0]),np.array([0,0,0.3,0]),np.array([0,0.2,0,0]),np.array([0,0,0,0.5])))


# Tests
def MinkowskiMetricTest(m,a,t,x,y,z,Kx,Ky,Kz,Jx,Jy,Jz):
    e0 = np.linalg.inv(Tetrad(m,a,t,x,y,z,Kx,Ky,Kz,Jx,Jy,Jz))
    g = KerrMetric(m,a,t,x,y,z,Kx,Ky,Kz,Jx,Jy,Jz)
    h = np.zeros((4,4))
    for u in range(0,4):
        for v in range(0,4):
            for a0 in range(0,4):
                for b in range(0,4):
                    h[u,v] = h[u,v] + g[a0,b]*e0[u,a0]*e0[v,b]
    eta0 = MinkowskiMetric()
    print(h-eta0)
def KerrMetricTest(m,a,t,x,y,z,Kx,Ky,Kz,Jx,Jy,Jz):
    X0 = np.array([t, x, y, z])
    delta0 = LorentzTransform(Kx, Ky, Kz, Jx, Jy, Jz)
    X = np.linalg.inv(delta0) @ X0
    x0 = X[1]
    y0 = X[2]
    z0 = X[3]
    N = MinkowskiMetric()
    r = radius(a,x0,y0,z0)
    H = Potential(m,a,r,z0)
    L = Vector(a,r,x0,y0,z0)
    # Construct Kerr Spacetime
    G = np.zeros((4,4))
    for u in range(0,4):
        for v in range(0,4):
            G[u,v] = N[u,v] + H*L[v]*L[u]
    G0 = KerrMetric(m,a,t,x,y,z,Kx,Ky,Kz,Jx,Jy,Jz)
    print(G-G0)
def GammaFrameTest():
    eta = MinkowskiMetric()
    I = np.eye(4)
    for mu in range(0,4):
        for nu in range(0,4):
```

```
            Amat = gamma[mu,:,:]
            Bmat = gamma[nu,:,:]
            Cmat = np.matmul(Amat,Bmat)
            Cmat0 = np.matmul(Bmat,Amat)
            mat = (Cmat + Cmat0)/2
            Imat0 = I * eta[mu,nu]
            print(mat-Imat0)
#GammaFrameTest()
def VolumeTest(m,a,t,x,y,z,Kx,Ky,Kz,Jx,Jy,Jz):
    e0 = Tetrad(m, a, t, x, y, z, Kx, Ky, Kz, Jx, Jy, Jz)
    G = KerrMetric(m, a, t, x, y, z, Kx, Ky, Kz, Jx, Jy, Jz)
    print(np.linalg.det(e0)-1)
    print(np.linalg.det(G)+1)
    print(np.linalg.det(T0)-1)
    print(np.linalg.det(X0)-1)
    print(np.linalg.det(Y0)-1)
    print(np.linalg.det(Z0)-1)
def LorentzTetradTest(m,a,t,x,y,z,Kx,Ky,Kz,Jx,Jy,Jz):
    G0 = (Tetrad(m,a,t,x,y,z,0,0,0,0,0,0))
    c0 = (LorentzTransform(Kx,Ky,Kz,Jx,Jy,Jz))
    X = c0 @ np.array([t,x,y,z])#@ np.linalg.inv(c0)
    cG0 = Tetrad(m,a,X[0],X[1],X[2],X[3],Kx,Ky,Kz,Jx,Jy,Jz)
    cG1 = G0 @ c0
    print(cG0-cG1)

def TestAll():
    m = np.random.rand()
    a = np.random.rand()
    t = np.random.rand()
    x = np.random.rand()
    y = np.random.rand()
    z = np.random.rand()
    Kx = np.random.rand()
    Ky = np.random.rand()
    Kz = np.random.rand()
    Jx = np.random.rand()
    Jy = np.random.rand()
    Jz = np.random.rand()
    MinkowskiMetricTest(m, a, t, x, y, z, Kx, Ky, Kz, Jx, Jy, Jz)
    LorentzTetradTest(m, a, t, x, y, z, Kx, Ky, Kz, Jx, Jy, Jz)
    KerrMetricTest(m, a, t, x, y, z, Kx, Ky, Kz, Jx, Jy, Jz)
    VolumeTest(m, a, t, x, y, z, Kx, Ky, Kz, Jx, Jy, Jz)
    GammaFrameTest()

TestAll()
```

My Grid Free Numerical Relativity "Black Hole Orbiter" Game:

```
#!/usr/bin/env python3
"""
Black Hole Orbiter (revised).

Improvements over original:
  1. Translated from MATLAB into Python, which opens the possibility of
     making a scientifically accurate game-engine with pygame visualizations
     while keeping low level control throughout for critical features.
  2. Simulates General Relativity Numerically, by referencing a 1975 paper
     "A Lorentz Covariant Treatment of the Kerr-Schild Geometry" by Gurses and Gursey.
     It is the only known way to fully simulate the General Theory of Relativity
     by using the only coordinate system that exactly linearizes the
     Einstein Field Equations, and does so without Supercomputing Clusters.
  3. Keeps the author's complete unifying vision of a Unified Field Theory intact.
     To do this requires a paradigm shift where every particle
     is also a rotating black hole, a field quantum, and an observer.
     This means one recovers Lorentz Covariant Motion in Special Relativity,
     Newtonian Force Laws of Universal Gravitation, Quantum Mechanics, and
     Multi-Particle Theory, with both mass and spin as particle invariants.

Author: Erik Jorgensen
Date: 2026-05-30 (revised), 2009-08-22 (original)
"""

import math
import numpy as np
from scipy.linalg import expm
import pygame
import sys

# ---------------------------------------------------------------------
# Minkowski metric  η_μν = diag(-1,+1,+1,+1)
# ---------------------------------------------------------------------

def minkowski_metric() -> np.ndarray:
    """Return η_μν with signature (−+++)."""
    return np.diag([-1.0, 1.0, 1.0, 1.0])


# ---------------------------------------------------------------------
# Change 4: Lorentz group via matrix exponential  (matches CGH.py)
# ---------------------------------------------------------------------

def lorentz_transform_matrix(Kx: float, Ky: float, Kz: float,
                             Jx: float, Jy: float, Jz: float) -> np.ndarray:
    """
    General Lorentz transformation  Λ = exp(M)  where M is the generator.

    K components are rapidities (boost parameters);
    J components are rotation angles.
    Convention identical to CGH.py:
        M = [[0,  Kx,  Ky,  Kz],
             [Kx,  0, -Jz,  Jy],
             [Ky,  Jz,  0, -Jx],
             [Kz, -Jy,  Jx,  0]]
    """
    M = np.array([[0.0,  Kx,   Ky,   Kz],
```

```
            [Kx,  0.0,  -Jz,   Jy],
            [Ky,   Jz,  0.0,  -Jx],
            [Kz,  -Jy,   Jx,  0.0]])
    return expm(M)


def boost_matrix_from_3vel(v3) -> np.ndarray:
    """
    Pure Lorentz boost for 3-velocity v3 = (vx, vy, vz).

    Converts to rapidity  η = arctanh(|v|)  and feeds the unit-direction
    components into lorentz_transform_matrix as (Kx, Ky, Kz).
    No rotation parameters are used.
    """
    vx, vy, vz = float(v3[0]), float(v3[1]), float(v3[2])
    vmag = math.sqrt(vx*vx + vy*vy + vz*vz)
    if vmag < 1e-14:
        return np.eye(4)
    vmag_c = min(vmag, 1.0 - 1e-10)   # clamp away from light-speed
    rapidity = math.atanh(vmag_c)
    nx, ny, nz = vx/vmag, vy/vmag, vz/vmag
    return lorentz_transform_matrix(nx*rapidity, ny*rapidity, nz*rapidity,
                                    0.0, 0.0, 0.0)


def lorentz_inverse(Lambda: np.ndarray) -> np.ndarray:
    """
    Inverse of a Lorentz matrix:  Λ^{-1} = η Λ^T η.
    Works for any element of O(1,3).
    """
    eta = minkowski_metric()
    return eta @ Lambda.T @ eta


def rotation_to_align_spin(spin_unit) -> tuple:
    """
    Return (R, R_inv): 4×4 spatial rotation matrices built from
    lorentz_transform_matrix (zero boost params) that actively rotate
    *spin_unit* onto +Z.

    Derivation
    ----------
    Given unit vector n̂ we need R such that  R n̂ = ẑ.
    Rotation axis:  k̂ = (n̂ × ẑ) / |n̂ × ẑ|
    Rotation angle: θ  = arccos(n̂ · ẑ)
    Rotation vector sent to the generator: ω = θ k̂

    With ẑ = (0,0,1):
        n̂ × ẑ = (ny, −nx, 0)   →   ω = θ(ny, −nx, 0) / sinθ

    The Jx, Jy, Jz parameters of lorentz_transform_matrix encode the
    rotation vector directly (no boost, K=0), so the result is a pure
    spatial rotation that leaves the time component untouched.

    Special cases
    -------------
    n̂ ≈ +ẑ : identity (no rotation needed).
    n̂ ≈ −ẑ : π rotation around x̂.
```

```
    """
    nx, ny, nz = float(spin_unit[0]), float(spin_unit[1]), float(spin_unit[2])
    cos_theta = max(-1.0, min(1.0, nz))   # n̂ · ẑ = nz

    if cos_theta > 1.0 - 1e-12:           # already aligned with +Z
        return np.eye(4), np.eye(4)

    if cos_theta < -1.0 + 1e-12:          # anti-aligned: rotate π around x̂
        R     = lorentz_transform_matrix(0, 0, 0, math.pi, 0, 0)
        R_inv = lorentz_transform_matrix(0, 0, 0, -math.pi, 0, 0)
        return R, R_inv

    theta     = math.acos(cos_theta)
    sin_theta = math.sqrt(max(1.0 - cos_theta*cos_theta, 0.0))

    # ω = θ (ny, −nx, 0) / sinθ
    omx =  theta * ny / sin_theta
    omy = -theta * nx / sin_theta
    omz = 0.0

    R     = lorentz_transform_matrix(0, 0, 0,  omx,  omy, omz)
    R_inv = lorentz_transform_matrix(0, 0, 0, -omx, -omy, omz)
    return R, R_inv

# ---------------------------------------------------------------------
# 4-vector / 4-velocity helpers
# ---------------------------------------------------------------------

def interval(v: np.ndarray) -> float:
    """Spacetime interval  η_μν v^μ v^ν."""
    return -v[0]*v[0] + v[1]*v[1] + v[2]*v[2] + v[3]*v[3]


def normalize_4velocity(u: np.ndarray) -> np.ndarray:
    """Rescale so that  η_μν u^μ u^ν = -1."""
    n2 = interval(u)
    if abs(n2) < 1e-14:
        return u
    return u / math.sqrt(max(-n2, 1e-14))

def normalize_4spin(j: np.ndarray) -> np.ndarray:
    """Rescale so that  η_μν j^μ j^ν = 1."""
    n2 = interval(j)
    if abs(n2) < 1e-14:
        return j
    return j / math.sqrt(max(n2, 1e-14))

def four_velocity_from_3velocity(v3) -> np.ndarray:
    vx, vy, vz = float(v3[0]), float(v3[1]), float(v3[2])
    vmag = math.sqrt(vx*vx + vy*vy + vz*vz)
    vmag = min(vmag, 1.0 - 1e-10)
    gamma = 1.0 / math.sqrt(1.0 - vmag*vmag)
    return np.array([gamma, gamma*vx, gamma*vy, gamma*vz])


def three_velocity_from_4velocity(u: np.ndarray) -> np.ndarray:
    if abs(u[0]) < 1e-14:
        return np.zeros(3)
```

```
    return u[1:4] / u[0]


# ---------------------------------------------------------------------
# Change 5: Kerr metric from Lorentz-covariant tetrad, no active transform
#
# The tetrad (from CGH.py) with all Lorentz parameters = 0:
#
#   e[μ, a] = δ[μ, a]  +  (H/2) L^μ  l_a
#
# where  l_a = η_{av} L^ν  and  L^μ is the Kerr-Schild null vector.
# Spin axis is always Z in the source rest frame.
# This is a purely coordinate description — no passive boosts or rotations
# are applied, which would not correspond to any physical transformation.
# ---------------------------------------------------------------------

def _ks_r(a: float, x: float, y: float, z: float) -> float:
    """Kerr radial coordinate r solving  r^4 - (R²-a²)r² - a²z² = 0."""
    R2 = x*x + y*y + z*z
    Ra = R2 - a*a
    b3 = math.sqrt(Ra*Ra + 4.0*a*a*z*z)
    r2 = (Ra + b3) * 0.5
    return math.sqrt(max(r2, 0.0))


def _ks_H(m: float, a: float, r: float, z: float) -> float:
    """
    Kerr-Schild scalar  H = 2 m r³ / (r⁴ + a² z²).
    Factor of 2 is absorbed here (matches CGH.py convention) so that
    g_μν = η_μν + H L_μ L_ν  (rather than  η_μν + 2V λ_μ λ_ν).
    """
    denom = r**4 + a*a*z*z
    if denom < 1e-30:
        return 0.0
    return 2.0 * m * r**3 / denom


def _ks_L(a: float, r: float, x: float, y: float, z: float) -> np.ndarray:
    """
    Kerr-Schild null vector  L^μ  (contravariant), spin along +Z.
    Null with respect to both η and g.
    """
    L = np.zeros(4)
    denom = r*r + a*a
    L[0] = 1.0
    if denom > 1e-30:
        L[1] = (r*x + a*y) / denom
        L[2] = (r*y - a*x) / denom
    if r > 1e-30:
        L[3] = z / r
    return L


def kerr_tetrad_restframe(m: float, a: float, x3) -> np.ndarray:
    """

    The tetrad  e[μ,a] = I[μ,a] + (H/2) L^μ (η_{av} L^ν)
    exactly as in CGH.py but with Lorentz parameters all zero (no active
```

```
    boost or rotation away from the standard Kerr-Schild chart):

    """
    x, y, z = float(x3[0]), float(x3[1]), float(x3[2])
    eta = minkowski_metric()
    r = _ks_r(a, x, y, z)
    H = _ks_H(m, a, r, z)
    L = _ks_L(a, r, x, y, z)        # L^μ  (upper)
    l = eta @ L                     # L_μ  (lower) = η_{μν} L^ν
    # Tetrad with Λ = I:  e[μ, a] = δ[μ,a] + (H/2) L[μ] l[a]
    e = np.eye(4) + 0.5 * H * np.outer(l, L)
    return e

def kerr_metric_restframe(m: float, a0: float, x3) -> np.ndarray:
    """
        Kerr metric g_μν at spatial position x3 = (x, y, z) in the source
        rest frame with spin axis along +Z.
              g[μ,ν] = η[a,b] e[μ,a] e[ν,b]
              = η[μ,ν] + H L[μ] L[ν]    (exploiting L·L = 0 w.r.t. η)

    The metric is stationary so only the spatial components of x3 matter.
    """
    # g[μ,ν] = η[a,b] e[μ,a] e[ν,b]
    eta = minkowski_metric()
    e = kerr_tetrad_restframe(m, a0, x3)
    return np.einsum('ab,ua,vb->uv', eta, e, e)

def christoffel_fd(x3, m: float, a: float, eps: float = 1e-6) -> np.ndarray:
    """
    Christoffel symbols  Γ^μ_{αβ}  via centred finite differences.
    The metric is stationary, so only spatial derivatives are non-zero.
    Returns shape (4, 4, 4):  Gamma[mu, alpha, beta].
    """
    g0 = kerr_metric_restframe(m, a, x3)
    ginv = np.linalg.inv(g0)

    # ∂_i g_{μν}  for i ∈ {x, y, z}  →  dg[spatial_index, μ, ν]
    dg_spatial = np.zeros((3, 4, 4))
    for i in range(3):
        dx = np.zeros(3)
        dx[i] = eps
        gp = kerr_metric_restframe(m, a, np.array(x3) + dx)
        gm = kerr_metric_restframe(m, a, np.array(x3) - dx)
        dg_spatial[i] = (gp - gm) / (2.0 * eps)

    # Full  ∂_α g_{μν}:  α=0 (time) is zero for stationary metric
    dg = np.zeros((4, 4, 4))        # dg[α, μ, ν]
    dg[1] = dg_spatial[0]
    dg[2] = dg_spatial[1]
    dg[3] = dg_spatial[2]

    # Γ^μ_{αβ} = ½ g^{μν} (∂_α g_{νβ} + ∂_β g_{να} − ∂_ν g_{αβ})
    Gamma = np.zeros((4, 4, 4))
    for mu in range(4):
        for alpha in range(4):
            for beta in range(4):
                s = 0.0
                for nu in range(4):
```

```
                s += ginv[mu, nu] * (dg[alpha, nu, beta]
                                   + dg[beta,  nu, alpha]
                                   - dg[nu, alpha, beta])
            Gamma[mu, alpha, beta] = 0.5 * s
    return Gamma


def geodesic_acceleration(u: np.ndarray, Gamma: np.ndarray) -> np.ndarray:
    """
    Geodesic 4-acceleration:  du^μ/dλ = −Γ^μ_{αβ} u^α u^β.
    """
    return -np.einsum('mab,a,b->m', Gamma, u, u)

def geodesic_spin_transport(j: np.ndarray, u: np.ndarray, Gamma: np.ndarray) -> np.ndarray:
    """
    Geodesic 4-spin parallel transport:  dj^μ/dλ = −Γ^μ_{αβ} j^α u^β.
    """
    return -np.einsum('mab,a,b->m', Gamma, j, u)

def Initialize_spin_vector(j: np.ndarray, u:np.ndarray):
    """
    Applies the physical constraint at t=0, such that
    j_μ u^μ = 0
    And it will evolve from there.
    """
    j[0] = (j[1]*u[1]+j[2]*u[2]+j[3]*u[3])/u[0]
    return j


# ---------------------------------------------------------------------
# Change 1 & 2: Particle class with worldline cache + proper-time state
# ---------------------------------------------------------------------

class Particle:
    """
    A gravitating body.  Integration is in proper time; coordinate time
    advances at rate  dt/dλ = u^0 (the Lorentz factor).

    The worldline cache stores snapshots at every integration step,
    ordered by (monotonically increasing) coordinate time.  Retarded
    positions are found by binary search on this cache.
    """

    def __init__(self, pos4: np.ndarray, vel4: np.ndarray, spin4: np.ndarray,
                 mass: float,
                 name: str = ""):
        """
        Parameters
        ----------
        pos4   : initial 4-position (t, x, y, z).
        vel4   : initial 4-velocity (will be normalised).
        spin4  : initial 4-spin     (will be normalised)
        mass   : gravitational mass (G = c = 1).
        spin_J : 3-vector whose *magnitude* is the Kerr spin parameter a
                 and whose *direction* is the spin axis in the lab frame.
                 Examples:
                   spin_J = (0, 0, 0.5)  -> a=0.5, spin along +Z
                   spin_J = (0.3, 0, 0)  -> a=0.3, spin along +X
```

```
    name   : optional label for diagnostics.
    """
    self.pos = pos4.astype(float).copy()
    self.vel = normalize_4velocity(vel4.astype(float).copy())
    self.spin = Initialize_spin_vector(spin4.astype(float).copy(),self.vel)
    self.mass = mass
    self.name = name

    self.proper_time: float = 0.0
    # Cache: list of dicts, sorted by ascending coordinate time.
    self._cache: list = [self._snapshot()]
    # ancestry tracking
    self.parent_ids = []  # names of particles that formed this one
    self.children = []  # particles formed from this one

    self.birth_time = None  # coordinate time at which this particle was born
    self.death_time = None          # coordinate time at which this particle died (None = still alive)
    self.is_ghost = False           # True once it has been merged away from the live list

# ------------------------------------------------------------------

def _snapshot(self) -> dict:
    return {
        't':   float(self.pos[0]),
        'pos': self.pos.copy(),
        'vel': self.vel.copy(),
        'spin': self.spin.copy(),
        'lam': self.proper_time,
    }

def record(self) -> None:
    """Append current state to cache."""
    self._cache.append(self._snapshot())

@property
def coord_time(self) -> float:
    return float(self.pos[0])

@property
def cache_t_min(self) -> float:
    return self._cache[0]['t']

@property
def cache_t_max(self) -> float:
    return self._cache[-1]['t']

# ------------------------------------------------------------------

def interpolate_at(self, t_query: float):
    """
    Linearly interpolate the cached worldline at coordinate time t_query.
    Returns (pos4, vel4).  Clamps to cache endpoints if out of range.
    """
    cache = self._cache
    if t_query <= cache[0]['t']:
        return cache[0]['pos'].copy(), cache[0]['vel'].copy(), cache[0]['spin'].copy(), cache[0]['lam']
    if t_query >= cache[-1]['t']:
        return cache[-1]['pos'].copy(), cache[-1]['vel'].copy(), cache[-1]['spin'].copy(), cache[-1]['lam']
```

```
    # Binary search for bracketing entries
    lo, hi = 0, len(cache) - 1
    while hi - lo > 1:
        mid = (lo + hi) >> 1
        if cache[mid]['t'] <= t_query:
            lo = mid
        else:
            hi = mid

    a0, a1 = cache[lo], cache[hi]
    dt = a1['t'] - a0['t']
    if dt < 1e-15:
        return a0['pos'].copy(), a0['vel'].copy(), a0['spin'].copy(), a0['lam']
    frac = (t_query - a0['t']) / dt
    pos = a0['pos'] + frac * (a1['pos'] - a0['pos'])
    vel = a0['vel'] + frac * (a1['vel'] - a0['vel'])
    spin = a0['spin'] + frac * (a1['spin'] - a0['spin'])
    lam =  a0['lam'] + frac * (a1['lam'] - a0['lam'])
    return pos, vel, spin, lam

# ------------------------------------------------------------------
# Change 1: retarded position via cache binary search
# ------------------------------------------------------------------

def retarded_position(self, observer_pos4: np.ndarray,
                      tol: float = 1e-10, max_iter: int = 64):
    """
    Find the retarded 4-position and 4-velocity of this particle as
    seen from observer_pos4 by binary-searching the worldline cache.

    Solves:  c·(t_obs − t_ret) = |x_src(t_ret) − x_obs|
             (with c = 1 in natural units)

    i.e. the separation 4-vector between source and observer is null.

    Parameters
    ----------
    observer_pos4 : 4-position of the observer at the *current* time.
    tol           : coordinate-time tolerance for the bisection.
    max_iter      : maximum bisection iterations.

    Returns
    -------
    pos_ret, vel_ret, spin_ret : 4-position, 4-velocity, 4-spin at the retarded event.
    """
    t_obs  = float(observer_pos4[0])
    x_obs  = observer_pos4[1:4]

    t_lo = self.cache_t_min
    t_hi = min(self.cache_t_max, t_obs)

    if t_hi <= t_lo:
        return self._cache[0]['pos'].copy(), self._cache[0]['vel'].copy(), self._cache[0]['spin'].copy(), self._cache[0]['lam']

    def residual(t_ret: float) -> float:
        """Positive when t_ret is too early, negative when too late."""
        pos_r, vel_r, spin_r, lam_r = self.interpolate_at(t_ret)
```

```
            dx = pos_r[1:4] - x_obs
            dist = math.sqrt(float(np.dot(dx, dx)))
            return (t_obs - t_ret) - dist

        f_lo = residual(t_lo)
        f_hi = residual(t_hi)

        # If the root is not bracketed, return the nearest endpoint
        if f_lo * f_hi > 0:
            t_ret = t_lo if abs(f_lo) < abs(f_hi) else t_hi
        else:
            for _ in range(max_iter):
                t_mid = 0.5 * (t_lo + t_hi)
                f_mid = residual(t_mid)
                if abs(f_mid) < tol:
                    break
                if f_lo * f_mid <= 0.0:
                    t_hi = t_mid
                    f_hi = f_mid
                else:
                    t_lo = t_mid
                    f_lo = f_mid
            t_ret = 0.5 * (t_lo + t_hi)

        return self.interpolate_at(t_ret)


# ---------------------------------------------------------------------
# N-body acceleration using retarded cache + frame-boosted geodesics
# ---------------------------------------------------------------------

def compute_acceleration(particle: Particle,
                         sources: list) -> np.ndarray:
    """
    Total geodesic 4-acceleration of *particle* due to all *sources*.

    For each source the evaluation chain is:

      lab  --boost-->  source rest frame  --rotate-->  Z-aligned frame
           <--un-boost--                  <--un-rotate--

      1. Retarded (causal) position/velocity from cache.
      2. Boost lab -> source rest frame  (matrix-exponential, K params).
      3. Active spatial rotation aligning src.spin_unit with +Z
         (matrix-exponential, J params only, K=0).
         This is sandwiched between the boosts; it is NOT applied to the
         tetrad, which would be a passive relabelling, not a physical rotation.
      4. Tetrad Kerr metric evaluated with spin along Z  (untouched).
      5. Geodesic acceleration in the Z-aligned source frame.
      6. Inverse rotation: Z-aligned -> source rest frame.
      7. Inverse boost: source rest frame -> lab frame.
    """
    du_total = np.zeros(4)
    dj_total = np.zeros(4)

    for src in sources:
        # 1. Retarded source state ----------------------------------------
        x_ret, u_ret, j_ret, lam_ret = src.retarded_position(particle.pos)
```

```
        v3_src = three_velocity_from_4velocity(u_ret)

        # 2. Boost: lab -> source rest frame --------------------------------
        Lambda_to_src   = boost_matrix_from_3vel(-v3_src)
        Lambda_from_src = lorentz_inverse(Lambda_to_src)

        u_src  = Lambda_to_src @ u_ret
        dx_src = Lambda_to_src @ (particle.pos - x_ret)
        j_src = Lambda_to_src @ j_ret

        # 3. Decompose spin_J into magnitude (spin parameter a) + unit direction
        norm_j_src = normalize_4spin(j_src)
        spin_a = math.sqrt(max(interval(j_src), 1e-14))
        if spin_a > 1e-14:
            spin_unit = (norm_j_src[1], norm_j_src[2], norm_j_src[3])
        else:
            spin_unit = np.array([0.0, 0.0, 1.0])


        # 4. Active rotation: source rest frame -> Z-aligned frame -----------
        # rotation_to_align_spin returns the matrix that takes spin_unit -> Z.
        # Both the position offset and the 4-velocity must be rotated so
        # that the Kerr geometry (whose symmetry axis is Z in the tetrad)
        # is evaluated in the correct orientation.

        R_to_z, R_from_z = rotation_to_align_spin(spin_unit)

        dx_rot = R_to_z @ dx_src
        u_rot  = R_to_z @ u_src
        j_rot  = R_to_z @ j_src
        x_rel  = dx_rot[1:4]          # spatial offset in Z-aligned frame

        # 5. Kerr geodesic in Z-aligned source frame (tetrad, untouched) -
        Gamma  = christoffel_fd(x_rel, src.mass, spin_a)
        du_rot = geodesic_acceleration(u_rot, Gamma)
        dj_rot = geodesic_spin_transport(j_rot,u_rot,Gamma)

        # 6. Un-rotate: Z-aligned -> source rest frame ----------------------
        du_src_frame = R_from_z @ du_rot
        dj_src_frame = R_from_z @ dj_rot

        # 7. Un-boost: source rest frame -> lab frame -----------------------
        du_total += Lambda_from_src @ du_src_frame
        dj_total += Lambda_from_src @ dj_src_frame

    return du_total, dj_total


def Quantum(particle: Particle, dlambda: float):
    M = particle.mass
    v4 = particle.vel
    v3 = three_velocity_from_4velocity(v4)
    dr = np.sqrt(dlambda/M)
    deltaX = np.zeros((4))
    deltaX[0] = 0
    deltaX[1] = np.random.normal() * dr
    deltaX[2] = np.random.normal() * dr
    deltaX[3] = np.random.normal() * dr
```

```
    Lambda_from_rest = boost_matrix_from_3vel(v3)
    dX4 = Lambda_from_rest @ deltaX
    return dX4


# ---------------------------------------------------------------------
# Change 2: proper-time integration step
# ---------------------------------------------------------------------

def euler_step(particle: Particle, dlambda: float,
               sources: list) -> None:
    """Advance particle by one proper-time step  dλ  (forward Euler)."""
    dX4 = Quantum(particle, dlambda)
    particle.pos += dX4
    du, dj = compute_acceleration(particle, sources)
    particle.vel = particle.vel + du * dlambda
    particle.spin = particle.spin + dj * dlambda
    particle.pos = particle.pos + particle.vel * dlambda
    particle.proper_time += dlambda
    particle.record()


# ---------------------------------------------------------------------
# Change 3: coordinate-time-synchronised N-body loop
#
# All particles are advanced in *their own proper times* until every
# particle's worldline cache extends to the same target coordinate time.
# This guarantees that the retarded-time binary search always has
# sufficient history for any particle pair, regardless of how fast each
# particle's coordinate time ticks relative to its proper time.
# ---------------------------------------------------------------------

def advance_to_coord_time(particle: Particle, t_target: float,
                          dlambda: float, sources: list,
                          integrator=euler_step) -> None:
    """
    Integrate *particle* in proper time until  coord_time >= t_target.

    The inner loop uses  dlambda  proper-time steps.  Because
        dt_coord / dλ = u^0 = γ ≥ 1
    particles moving rapidly will take *fewer* proper-time steps to
    reach the same coordinate-time target.  The cache accumulates all
    intermediate states, providing a dense worldline for retarded lookups.
    """
    while particle.coord_time < t_target:
        integrator(particle, dlambda, sources)



# ---------------------------------------------------------------------
# FIX: build_observer_view takes particles (with caches), not histories
# ---------------------------------------------------------------------
def build_observer_view(observer: Particle,
                        particles: list,
                        ghosts: list = None):
    """
    Build a first-person view for `observer` at its current event.
```

```
*For each source:*
*  - Find the retarded event on the source worldline as seen by the observer.*
*  - Form the separation 4-vector in the lab frame.*
*  - Boost into the observer's instantaneous rest frame.*
*  - Rotate so that the observer's spin points along +Z.*
*Returns a list of dicts, one per visible source.*
"""
"""
    Build a first-person view for `observer` at its current event.

    Includes both live particles and ghosts.  A ghost is only visible
    while the retarded time the observer is seeing is still earlier
    than that ghost's death_time.  This keeps the pre-merger particles
    on screen until the light from the merger reaches the camera.

    All returned data are already in the observer's instantaneous
    rest frame (boosted + spin-aligned).  No lab-frame quantities
    are exposed to the renderer.
"""
if ghosts is None:
    ghosts = []

# Observer state in lab frame
x_obs = observer.pos.copy()
u_obs = observer.vel.copy()
j_obs = observer.spin.copy()

# 3-velocity of observer
v3_obs = three_velocity_from_4velocity(u_obs)

# Boost lab -> observer rest frame
Lambda_to_obs   = boost_matrix_from_3vel(-v3_obs)
Lambda_from_obs = lorentz_inverse(Lambda_to_obs)

# Observer spin direction in its rest frame (for orientation)
j_obs_rest = Lambda_to_obs @ j_obs
j_obs_rest = normalize_4spin(j_obs_rest)
spin_unit_obs = np.array([j_obs_rest[1], j_obs_rest[2], j_obs_rest[3]])

# Rotate so that observer spin points along +Z
R_to_z, R_from_z = rotation_to_align_spin(spin_unit_obs)

events = []

# Candidates = live particles + ghosts
candidates = list(particles) + list(ghosts)

for src in candidates:
    if src is observer:
        continue

    # Retarded event of source as seen from observer's current position
    x_ret, u_ret, j_ret, lam_ret = src.retarded_position(x_obs)

    t_ret = float(x_ret[0])
    # ---------- visibility window ----------
    # Birth: particle is invisible until the light from its birth event
    # actually reaches the observer.  A clamped t_ret at the start of
```

```
        # the cache means the light has not arrived yet.
        if src.birth_time is not None:
            if t_ret < src.birth_time + 1e-8:
                continue
            # Still clamped to the very first cache entry → light has not arrived
            if abs(t_ret - src.cache_t_min) < 1e-8:
                continue

        # Death (ghosts): disappear once retarded time reaches death event
        # or the cache has ended
        if src.death_time is not None:
            if t_ret >= src.death_time - 1e-6:
                continue
            if t_ret >= src.cache_t_max - 1e-6:
                continue
        # ----------------------------------------

        # Separation in lab frame: source_ret - observer_now
        dx_lab = x_ret - x_obs

        # Boost into observer rest frame
        dx_obs = Lambda_to_obs @ dx_lab

        # Rotate so that observer spin is +Z
        dx_obs_z = R_to_z @ dx_obs

        events.append({
            'name': src.name,
            'x4_obs': dx_obs,      # 4-position of source event in observer Z-aligned frame
            'lambda_src_ret': lam_ret,
            'src_mass': src.mass,
            'is_ghost': src.is_ghost,
            'death_time': src.death_time,
            't_ret': t_ret,
        })

    return events

def run_nbody_step(particles: list,
                   ghosts: list,
                   observer_name: str,
                   dlambda: float,
                   frame_idx: int,
                   print_every: int = 10):
    """
    Performs ONE observer-driven N-body frame step.

    Returns
    -------
    view_events : list of retarded, boosted, rotated source events
                  (already player-centric; includes visible ghosts)
    t_obs       : observer coordinate time
    particles   : (possibly updated) live particle list
    ghosts      : updated ghost list
    """

    particles_by_name = {p.name: p for p in particles}
    observer = particles_by_name[observer_name]
```

```
    # 1. Advance the observer one proper-time step
    other_particles = [p for p in particles if p is not observer]
    euler_step(observer, dlambda, other_particles)

    # 2. Target coordinate time defined by the observer
    t_target = observer.coord_time

    # 3. Advance all other live particles to the same coordinate time
    for p in other_particles:
        p_sources = [q for q in particles if q is not p]
        advance_to_coord_time(p, t_target, dlambda, p_sources)

    # 4. Build the player-centric view (live + still-visible ghosts)
    view_events = build_observer_view(observer, particles, ghosts)

    # 5. Print observer-view data (player-centric only)
    return view_events, t_target, particles, ghosts


def spawn_grid(n, spacing):
    pts = []
    counter = 0

    for i in range(n):
        x = np.random.normal()*spacing
        y = np.random.normal()*spacing
        z = np.random.normal()*spacing

        pos4 = np.array([0.0, x, y, z])
        vel4 = four_velocity_from_3velocity((0,0,0))

        spin = np.array([
            0.0,
            np.random.normal(),
            np.random.normal(),
            np.random.normal()
        ])

        pts.append(
            Particle(
                pos4=pos4,
                vel4=vel4,
                spin4=spin,
                mass=np.random.uniform(),
                name=f"p{counter}"
            )
        )
        counter += 1

    return pts

def run_example_debug(n,r):
    particles = []
    # add grid particles
    particles += spawn_grid(n, r)
    observer = particles[0]  # or whichever
    observer_name = observer.name
```

```
    dlambda = 1
    return particles, observer_name, dlambda

def run_example():
    M = 10
    r0 = 100
    v_c = math.sqrt(M / r0)

    p1 = Particle(
        pos4=np.array([0.0, 0.0, -r0, 0.0]),
        vel4=four_velocity_from_3velocity((v_c / 2, 0.0, 0.0)),
        mass=M,
        spin4=np.array([0.0, 0.5, 0.0, 0.0]),
        name="p1",
    )
    p1.birth_time = p1.pos[0]
    p2 = Particle(
        pos4=np.array([0.0, 0.0, r0, 0.0]),
        vel4=four_velocity_from_3velocity((-v_c / 2, 0.0, 0.0)),
        mass=M,
        spin4=np.array([0.0, -0.4, 0.0, 0.3]),
        name="p2",
    )
    p2.birth_time = p2.pos[0]
    p3 = Particle(
        pos4=np.array([0.0, 0, 0, -r0*15]),
        vel4=four_velocity_from_3velocity((0.0, 0.0, -0.1)),
        mass=M,
        spin4=np.array([0.0, 0.0, 0.0, 1.0]),
        name="p3",
    )
    p3.birth_time = p3.pos[0]

    particles = [p1, p2, p3]
    observer_name = "p3"
    dlambda = 10

    return particles, observer_name, dlambda

def rebind_observer(observer, particles):
    # Case 1: observer still exists
    if observer in particles:
        return observer

    # Case 2: observer was merged — find the particle that lists it as a parent
    for p in particles:
        if observer.name in p.parent_ids:
            return p

    # Case 3: fallback — choose nearest particle in spacetime
    return min(
        particles,
        key=lambda p: np.linalg.norm(p.pos4 - observer.pos4)
    )

def relative_ks_r(p_obs: Particle, p_src: Particle) -> float:
    """
    Kerr radial coordinate of p_obs as measured in the instantaneous
```

```
    rest frame of p_src after spin alignment to +Z.
    Exactly the same boost → rotate sandwich used by compute_acceleration.
    """
    # Use current states (pair is already synchronized and close)
    x_ret0, u_ret0, j_ret0, lam_ret0 = p_src.retarded_position(p_obs.pos.copy())
    v3_src = three_velocity_from_4velocity(u_ret0)
    Lambda_to_src = boost_matrix_from_3vel(-v3_src)

    dx_src = Lambda_to_src @ (p_obs.pos - x_ret0)

    j_src = Lambda_to_src @ j_ret0
    spin_a = math.sqrt(max(interval(j_src), 1e-14))
    if spin_a > 1e-14:
        norm_j = normalize_4spin(j_src)
        spin_unit = (norm_j[1], norm_j[2], norm_j[3])
    else:
        spin_unit = np.array([0.0, 0.0, 1.0])

    R_to_z, _ = rotation_to_align_spin(spin_unit)
    dx_rot = R_to_z @ dx_src
    x_rel = dx_rot[1:4]

    return _ks_r(spin_a, x_rel[0], x_rel[1], x_rel[2])

def is_singularity(p1: Particle, p2: Particle, eps: float = 1) -> bool:
    """
    True when the relative Kerr radius of either particle inside the
    other has fallen below eps.  This is the point at which the
    finite-difference Christoffel symbols become unreliable.
    """
    r12 = relative_ks_r(p1, p2)
    r21 = relative_ks_r(p2, p1)
    return min(r12, r21) < eps

def renorm_via_tetrad(V_lab: np.ndarray,
                      src: Particle,
                      obs_pos: np.ndarray) -> np.ndarray:
    """
    Frame sandwich identical to compute_acceleration, but the final
    operation is a tetrad projection instead of a geodesic acceleration.

    lab  --boost-->  src rest  --rotate-->  Z-aligned
        <--un-boost--          <--un-rotate--

    Inside the Z-aligned chart the (copy of the) rest-frame tetrad
    converts the coordinate 4-vector into a local Lorentz (“flat”)
    4-vector.  The inverse sandwich then returns that local vector
    to the lab.  A transpose of the *copy* is used if needed; the
    original tetrad function is never modified.
    """
    v3_src = three_velocity_from_4velocity(src.vel)
    Lambda_to_src   = boost_matrix_from_3vel(-v3_src)
    Lambda_from_src = lorentz_inverse(Lambda_to_src)

    V_src  = Lambda_to_src @ V_lab
    dx_src = Lambda_to_src @ (obs_pos - src.pos)

    j_src = Lambda_to_src @ src.spin
```

```
    spin_a = math.sqrt(max(interval(j_src), 1e-14))
    if spin_a > 1e-14:
        norm_j = normalize_4spin(j_src)
        spin_unit = (norm_j[1], norm_j[2], norm_j[3])
    else:
        spin_unit = np.array([0.0, 0.0, 1.0])

    R_to_z, R_from_z = rotation_to_align_spin(spin_unit)

    V_rot  = R_to_z @ V_src
    dx_rot = R_to_z @ dx_src
    x_rel  = dx_rot[1:4]

    # --- tetrad in the compute frame (copy only) ---
    e = kerr_tetrad_restframe(src.mass, spin_a, x_rel).copy()

    # Optional transpose of the *copy* (never of the original function).
    # Try e.T first; if mass explodes, switch to np.linalg.inv(e).
    # Both preserve the metric factors that encode time dilation.
    e_map = e          # or np.linalg.inv(e) if preferred after testing

    # Coordinate → local Lorentz
    V_local = e_map @ V_rot

    # --- return to lab ---
    V_src_frame = R_from_z @ V_local
    V_lab_ren   = Lambda_from_src @ V_src_frame
    return V_lab_ren

def merge_particles_tetrad(p1: Particle, p2: Particle) -> Particle:
    """
    Replace the naïve P1+P2 addition by the two-sided tetrad
    renormalization described in the request.

    1. Map (P1, J1) through p2's geometry → contribution of parent 1
    2. Map (P2, J2) through p1's geometry → contribution of parent 2
    3. Sum the renormalized vectors
    4. Reconstruct mass, 4-velocity and 4-spin from the sum
    """
    # Lab 4-momenta and 4-spins (identical to original merge_particles)
    P1 = p1.mass * p1.vel
    P2 = p2.mass * p2.vel
    J1 = p1.spin.copy()
    J2 = p2.spin.copy()

    # One-sided renormalizations
    P1_ren = renorm_via_tetrad(P1, p2, p1.pos)
    J1_ren = renorm_via_tetrad(J1, p2, p1.pos)

    P2_ren = renorm_via_tetrad(P2, p1, p2.pos)
    J2_ren = renorm_via_tetrad(J2, p1, p2.pos)

    # Sum (the tetrad factors have already suppressed the near-singularity blow-up)
    P_new = P1_ren + P2_ren
    J_new = J1_ren + J2_ren

    m_new = math.sqrt(max(-(interval(P_new)), 1e-14))
    u_new = P_new / m_new
```

```
    # Position still the momentum-weighted average (or singularity location)
    x_new = (p1.mass * p1.pos + p2.mass * p2.pos) / (p1.mass + p2.mass)

    merged = Particle(
        pos4=x_new,
        vel4=u_new,
        spin4=J_new,
        mass=m_new,
        name=f"{p1.name}+{p2.name}"
    )
    merged.parent_ids = [p1.name, p2.name]
    p1.children.append(merged)
    p2.children.append(merged)
    return merged

def horizon_radius(p: Particle) -> float:
    M = 2*p.mass
    return M

def check_merger_condition(p1: Particle, p2: Particle) -> bool:
    """
    Simple geometric merger rule:
    two holes merge when their spatial separation
    is less than the sum of their horizon radii.
    """
    x1 = p1.pos[1:4]
    x2 = p2.pos[1:4]
    r12 = float(np.linalg.norm(x1 - x2))

    R1 = horizon_radius(p1)
    R2 = horizon_radius(p2)

    return r12 <= (R1 + R2)

def apply_mergers(particles, ghosts, dlambda_plunge=1, max_plunge_steps=2000):
    """
    Keep original horizon test as the trigger that a pair has entered
    the strong-field regime.  Once triggered, integrate the pair in
    isolation until a singularity is reached, discarding any cache
    entries that were written after the singularity time.  Only then
    perform the tetrad-renormalized merger.
    """
    new_live = []
    new_ghosts = list(ghosts)
    skip = set()

    for i, p1 in enumerate(particles):
        if i in skip:
            continue
        for j, p2 in enumerate(particles):
            if j <= i or j in skip:
                continue

            # ----- original geometric horizon test (entry condition) -----
            if not check_merger_condition(p1, p2):
                continue
```

```
        # ----- plunge until singularity -----
        singularity_hit = False
        for _ in range(max_plunge_steps):
            if is_singularity(p1, p2):
                singularity_hit = True
                break

            # Guarded Euler steps (only the mutual interaction)
            try:
                # record length before the step so we can discard
                len1 = len(p1._cache)
                len2 = len(p2._cache)

                euler_step(p1, dlambda_plunge, [p2])
                euler_step(p2, dlambda_plunge, [p1])

                # crude NaN / explosion guard
                if (np.linalg.norm(p1.pos[1:4]-p2.pos[1:4]) > (p1.mass + p2.mass) or
                    abs(p1.vel[0]) > 100 or abs(p2.vel[0]) > 100 or
                    not np.all(np.isfinite(p1.pos)) or not np.all(np.isfinite(p2.pos))):
                    # discard the bad entries
                    p1._cache = p1._cache[:len1]
                    p2._cache = p2._cache[:len2]
                    # restore last good state
                    if len1 > 0:
                        last = p1._cache[-1]
                        p1.pos, p1.vel, p1.spin = last['pos'], last['vel'], last['spin']
                        p1.proper_time = last['lam']
                    if len2 > 0:
                        last = p2._cache[-1]
                        p2.pos, p2.vel, p2.spin = last['pos'], last['vel'], last['spin']
                        p2.proper_time = last['lam']
                    singularity_hit = True
                    break
            except Exception:
                # any numerical failure → treat as singularity
                p1._cache = p1._cache[:len1]
                p2._cache = p2._cache[:len2]
                singularity_hit = True
                break

        if not singularity_hit:
            # safety: force merge after max steps
            singularity_hit = True

        # ----- singularity reached: tetrad merger -----
        t_merge = max(p1.coord_time, p2.coord_time)

        merged = merge_particles_tetrad(p1, p2)

        merged.birth_time = t_merge

        p1.death_time = t_merge
        p2.death_time = t_merge
        p1.is_ghost = True
        p2.is_ghost = True

        new_ghosts.append(p1)
```

```
            new_ghosts.append(p2)
            new_live.append(merged)

            skip.add(i)
            skip.add(j)
            break
        else:
            if not p1.is_ghost:
                new_live.append(p1)

    return new_live, new_ghosts


def prune_ghosts(ghosts, observer):
    """Remove ghosts whose light has already reached the current observer."""
    alive_ghosts = []
    x_obs = observer.pos
    for g in ghosts:
        if g.death_time is None:
            alive_ghosts.append(g)
            continue
        # Quick check: if the latest possible retarded time is already past death
        if g.cache_t_max < g.death_time - 1e-6:
            # cache ended before death_time – keep for a moment longer if needed
            alive_ghosts.append(g)
            continue
        # Ask for the retarded time the observer would see right now
        x_ret, _, _, _ = g.retarded_position(x_obs)
        t_ret = float(x_ret[0])
        if t_ret < g.death_time - 1e-6 and t_ret < g.cache_t_max - 1e-6:
            alive_ghosts.append(g)
        # otherwise the ghost is no longer visible → drop it
    return alive_ghosts

    # ------------------------------------------------------------------
    # MAIN - CLEAN BODY TRIAD VERSION (only this section is improved)
    # ------------------------------------------------------------------

def main():
    pygame.init()
    width, height = 720, 480
    screen = pygame.display.set_mode((width, height))
    pygame.display.set_caption("Black Hole Orbiter - Body Frame Camera")
    clock = pygame.time.Clock()

    particles, observer_name, dlambda = run_example()
    ghosts = []
    frame_idx = 0
    observer = {p.name: p for p in particles}[observer_name]

    prev_lambdas = {}  # name → last λ_ret the observer saw

    # Body triad (world space)
    w_body = np.array([0,0,0],dtype=float)
    # Convention: X = right, Y = up, Z = forward  (matches Godot feel + Python depth = component 3)
    X = np.array([1.0, 0.0, 0.0])
    Y = np.array([0.0, 0.0, 1.0])
    Z = np.array([0.0, 1.0, 0.0])
```

```
R0 = np.eye(4)

boost_strength = 0.06
rot_strength   = 0.001

running = True
while running:
    for event in pygame.event.get():
        if event.type == pygame.QUIT:
            running = False

    keys = pygame.key.get_pressed()

    # ---------- Body-frame input ----------
    boost_input = np.zeros(3)
    rot_input   = np.zeros(3)

    w_world = X * w_body[0] + Y * w_body[1] + Z * w_body[2]

    # Pure spatial rotation (your preferred method)
    R = lorentz_transform_matrix(0.0, 0.0, 0.0,
                                 w_world[0], w_world[1], w_world[2])

    # Rotate the triad (extract the spatial 3×3 block)
    R0 = R0 @ R
    R3 = np.linalg.inv(R[1:4,1:4])

    # Movement (WASD + Space/Shift)  – body frame
    if keys[pygame.K_w]:         boost_input[1] += 1   # Forward
    if keys[pygame.K_s]:         boost_input[1] -= 1   # Backward
    if keys[pygame.K_a]:         boost_input[0] -= 1   # Left
    if keys[pygame.K_d]:         boost_input[0] += 1   # Right
    if keys[pygame.K_SPACE]:     boost_input[2] += 1   # Up
    if keys[pygame.K_LSHIFT] or keys[pygame.K_RSHIFT]:
                                 boost_input[2] -= 1   # Down

    # Rotation (Arrow keys + Q/E)  – body frame
    if keys[pygame.K_UP]:        rot_input[0] += 1     # Pitch up
    if keys[pygame.K_DOWN]:      rot_input[0] -= 1     # Pitch down
    if keys[pygame.K_LEFT]:      rot_input[2] += 1     # Yaw left
    if keys[pygame.K_RIGHT]:     rot_input[2] -= 1     # Yaw right
    if keys[pygame.K_q]:         rot_input[1] += 1     # Roll left
    if keys[pygame.K_e]:         rot_input[1] -= 1     # Roll right

    # ---------- Body-frame boost ----------
    if np.linalg.norm(boost_input) > 1e-6:
        body_dir = boost_input / np.linalg.norm(boost_input)
        world_dir = body_dir[0]*X + body_dir[1]*Y + body_dir[2]*Z
        Boost = boost_matrix_from_3vel(world_dir * boost_strength)
        observer.vel  = Boost @ observer.vel
        observer.spin = Boost @ observer.spin

    # ---------- Body-frame rotation using your lorentz_transform_matrix ----------
    if np.linalg.norm(rot_input) > 1e-6:
        # Body-frame generator components
        body_dir0 = rot_input / np.linalg.norm(rot_input)
        #world_dir0 = body_dir0[0]*X + body_dir0[1]*Y + body_dir0[2]*Z
```

```
    w_body += body_dir0 * rot_strength


#R3 =
X = R3 @ X
Y = R3 @ Y
Z = R3 @ Z

# Generate the spin from the observer's angular velocity
observer.spin = np.array((0.0, w_world[0], w_world[1], w_world[2]))

# Final constraints
observer.vel  = normalize_4velocity(observer.vel)
observer.spin = Initialize_spin_vector(observer.spin, observer.vel)

# ---------- Physics (unchanged) ----------
view_events, t_obs, particles, ghosts = run_nbody_step(
    particles, ghosts, observer_name, dlambda, frame_idx
)

# Apply mergers (returns both live list and updated ghosts)
particles, ghosts = apply_mergers(particles, ghosts)

observer = rebind_observer(observer, particles)
observer_name = observer.name

ghosts = prune_ghosts(ghosts, observer)

# ---------- Render ----------
screen.fill((0, 0, 0))

for ev in view_events:
    name = ev['name']
    lam = ev['lambda_src_ret']

    # Δλ since the last time this source was seen by the observer
    if name in prev_lambdas:
        delta_lambda = lam - prev_lambdas[name]
    else:
        delta_lambda = 0.0  # first appearance

    # Store for next frame
    prev_lambdas[name] = lam

    # Safety: ignore negative or tiny values (can happen with ghosts / clamping)
    if delta_lambda < 1e-8:
        delta_lambda = 1e-8

    # ----- brightness model -----
    # Simple linear scaling with a soft clamp.
    # You can later replace this with any function of Δλ.
    brightness = min(1.0, delta_lambda * 0.5 / dlambda)  # tune the 3.0 factor
    # brightness is now in [0.15 … 1.0]

    grey = int(255 * brightness)
    color = (grey, grey, grey)
    size_factor = 1
```

```
            x4 = R0 @ ev["x4_obs"]
            Xv, Yv, Zv = x4[1], x4[2], x4[3]

            if Zv > 0.1:
                sx = int(width / 2 + 540 * Xv / Zv)
                sy = int(height / 2 - 540 * Yv / Zv)

                if 0 <= sx < width and 0 <= sy < height:
                    dist = max(np.sqrt(Xv * Xv + Yv * Yv + Zv * Zv), 1.0)
                    radius = max(1, int(540 * ev['src_mass'] * size_factor / dist))
                    pygame.draw.circle(screen, color, (sx, sy), radius)

        # Clean up prev_lambdas for particles that are no longer visible
        # (optional but keeps the dict from growing forever)
        visible_names = {ev['name'] for ev in view_events}
        prev_lambdas = {k: v for k, v in prev_lambdas.items() if k in visible_names}
        pygame.display.flip()
        clock.tick(60)
        frame_idx += 1
    pygame.quit()
    sys.exit()

if __name__ == "__main__":
    main()
```